Dynamic Semantics and the Geometry of Meaning

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"The opposite of a profound truth may well be another profound truth." (Niels Bohr)

1 As prominently demonstrated in theoretical physics, the formal language of mathematics may be very useful for describing aspects of reality. That does not mean that the mathematical instruments are intended to capture a precise picture of reality. Instead, processes of abstraction and idealization are omnipresent generating an apparently very close fit between preexisting, pleasant mathematical structures and an idealized/abstracted picture of reality, which is studied in science.

In a target article published in *Theoretical Linguistics*, Stokhof and van Lambalgen [42] analyzed the role of idealizations and abstractions as part of "the construction of modern linguistics." In the present short note we develop some ideas which were presented earlier as a comment on this article [10]. The main tenet of the target article was that (most) parts of modern linguistics are based on the idea of idealization rather than the idea of abstraction, which the authors consider as more useful. Even when the authors address the 'Generative tradition', in most parts of their analysis they do not exclude other developments in semantics and pragmatics:

> That being said, although we did not explicitly argue this to any extent in the paper, we are of the opinion that some of the basic constructions that originated with the generative tradition in syntax continue to be operative also in other paradigms and other linguistic disciplines. And this should not come as a surprise of course. For one thing, many such paradigms are explicitly developed as alternatives to the generative one, and no alternative does not share some basic assumptions and concepts with the original it wants to supersede. Also, the diversity that is characteristic of present-day linguistics is the result of the extension of methods and concepts that were originally developed in syntax to other branches of linguistics, such as semantics and pragmatics. Thus we think it is fairly obvious that despite the many and important differences that exist between various alternative paradigms, these also share a number of core assumptions and concepts with the generative tradition. And in as much as these assumptions and concepts are instances of what we have labelled 'idealisation', that means that our analysis in effect extends to these alternative paradigms as well, and thus that, appearances notwithstanding, it does range over more than just the generative tradition in modern linguistics even if we take the latter term to refer to present-day linguistics. [43, pp. 80ff]

In this short note, we will apply their ideas to the domain of semantics. A basic question of formal semantics is "what are propositions?" Since the beginning of formal logic and natural language semantics there seems to be a far-reaching agreement that propositions are to be considered as entities that form a Boolean lattice. We think that this assumption is arguably an instance of idealization in the domain of semantics.

In order to prepare our answer to the basic question above, we start with the distinction of two different types of questions: factual questions and attitude questions. This distinction is based on modern survey research (e.g. [45, 37]). In a factual question the interviewer typically asks the respondent about her personal activities or circumstances. In attitude questions, by contrast, the interviewer seeks the respondent's opinion about an issue. So far as we can see, logical-semantic analyses of questions have almost exclusively concentrated on factual questions. Despite their practical importance attitude questions have mostly been ignored in the literature of formal semantics.

Why are attitude questions so interesting? Survey researchers have demonstrated repeatedly that the same question often produces quite different answers, depending on the question context (for numerous survey examples, see [44, 37]. To cite just one particularly well-documented example, a group of (North-American) subjects were asked whether "the United States should let Communist reporters come in here and send back to their papers the news as they see it?" The other group was asked whether "a Communist country like Russia should let American newspaper reporters come in and send back to their papers the news as they see it?" Support for free access for the Communist reporters varied sharply according to whether that question preceded or followed the question on American reporters. The differences are quite dramatic: in a study of 1950, 36% accepted communist reporters when the communist question came first and 73% accepted them when the question came second. When the study was repeated in 1982, the numbers changed to 55% vs. 75%. This example illustrates the *non-commutative* character of attitude questions — a feature typically found for attitude questions but not for factual questions. The existence of attitude questions and the observed order effects exclude the analysis of their truth-conditional content in terms of Boolean lattices. Hence, in accepting Boolean structure, we ignore the order effects that are essential for the semantics of attitude questions. This is an example of idealization since some crucial qualitative feature is missing after doing this simplification. It is the feature of *non-commutativity* (also called *complementarity*) that is missing now, a feature that can be seen as responsible for the observed order effects [15, 13].

What is the alternative that comprises both factual and attitude questions? The proposal is to consider so-called *orthomodular lattices* [6, 7, 11]. These lattices account for the observed order effects for attitude questions and they are compatible with the commutativity of questions in case of factual questions. Assuming that propositions form an orthomodular lattice is an abstraction. It takes all types of propositions (and questions) into account and abstracts from marginal features such as those determining representativeness and typicality. Technically, orthomodular lattices can be understood as the union on several blocks of (partial) Boolean lattices. If such blocks are brought together by some kind of unification, the resulting lattice can violate distributivity. As a consequence, such structures allow the expression of ordering effects as investigated in survey research. The probability that a proposition is considered true can depend on former question material. Further, many puzzles known from the context of bounded rationality can be handled with the abstraction of considering propositions as elements of a orthomodular lattice [15, 16]. All these puzzles cannot be handled if one resorts to Boolean idealization.

2 As former students of physics the authors of this squib share the view that empirical science is more than the collection of data and the development of a theory that is able to fit a substantial part of the data. Rather, the development of theories is assumed to lead to an understanding of nature. We think that this idea also applies to theoretical linguistics in general and formal semantics/pragmatics in particular. In the field of natural language semantics and pragmatics, some researchers take a similar naturalistic stance and claim that basic prin-

ciples of cognitive psychology can be applied for grounding the basic mechanisms of natural language interpretation. The view of placing natural language semantics/pragmatics within the scope of a naturalistic (explanatory) approach is not without problems. This has to do with the normative character that is attributed to a Gricean setting of action-based linguistics. Speakers, as Grice [23, p. 45] puts it, must "make their contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which (they) are engaged." Obviously, this Gricean principle of cooperation is normative (and so are Grice's conversational maxims). Even though the normative and the naturalistic aspects of understanding human actions can be clearly separated from each other, in most cases it does not follow that they predict different action patterns. The idea of a rational world is not so irrational to be excluded in ordinary affairs [12]. Evolutionary game theory has presented us with many examples demonstrating that the reasonable is naturally arising [5]. In other words, though there is a philosophical gap between normative theories of pragmasemantics and theoretical frameworks as scientific, explanatory theories of natural language, there is not a deep empirical conflict between both approaches (for a similar conclusion, see [29]).

Finding proper explanations instead of pure descriptions crucially hangs on the correct use of abstractions rather than idealizations. If this is true, then the idea of using orthomodular lattices instead of Boolean algebras for the modelling of propositions may be a fundamental assumption of semantics that should be combined with other fundamental insights of formal semantics.

3 Martin Stokhof, Jeroen Groenendijk, and Frank Veltman initiated the disciplines of dynamic semantics, epistemic logics and pragmatic information theory — together with other researchers such as Peter Gärdenfors [18] and Johan van Benthem [46]. According to this approach, the meaning of a proposition is not its truth value or a set of possible worlds, but rather its impact upon the epistemic state of a cognitive agent. The triumvirate the present *festschrift* is devoted to has made considerable contributions to the field of dynamic semantics and update semantics, with applications improving our understanding of a whole area of phenomena ranging from epistemic modals and counterfactuals [49], to inferential pattern of generics [48], discourse phenomena [27], and including the field of the semantics of questions and answers [24, 25, 26, 28].

In their analysis, the authors were able to give an analysis of certain order effects even without giving up the idealization of Boolean propositions. One important example for order effects in dynamic semantics is the resolution of anaphors. For this aim, Staudacher [41] and Groenendijk and Stokhof [27] have independently developed models of dynamic predicate logics, where quantifiers, such as 'there exists an x' or 'for all x', and anaphors, e.g. pronouns, are described as certain operators acting upon model theoretic valuations. And Veltman [48] has observed that by relaxing the stability conditions of logical consequences, dynamic logics becomes non-monotonic. This allows the treatment of default operations, such as "may" or "normally". Veltman [48] presented a fine example for such an ordering effect in default reasoning: Let A = "somebody is knocking at the door", B = "Maybe it's John", and C = "Its Mary". Then the composition CBA = "Somebody is knocking at the door. Maybe it's John. It's Mary" makes perfect sense, while BCBA = "Somebody is knocking at the door. Maybe it's John. It's Mary. Maybe it's John" does not.

4 Taking the fundamental criticism of Stokhof and van Lambalgen [42] as a starting point, we would like to make a suggestion for a research program that takes up some challenges of their analysis in the field of semantics and pragmatics. The proposal is to unify the existing proposals of dynamic semantics and update semantics with a proper analysis of propositions in terms of

orthomodular lattices.[1] The surprising result is that this treatment can also account for the order effects of attitude questions [50], many effects of contextuality and bounded rationality [15], and a systematic treatment of belief-revision in the sense of Gärdenfors [18] — for the latter see [20, 22]. The key idea that allows the unification of non-commutative belief-revision, Bayesian update semantics, and dynamic semantics for discourse phenomena is the exhibition of formal analogies to generalized quantum theory and algebraic representation theory. In generalized quantum theory, observations are expressed as testing states [6, 7], which does not necessarily lead to commutative operations. In algebraic representation theory the algebraic structures of observations become represented by matrices acting on vector spaces. In this sense, the interpretation functions of dynamic semantics 'represent' the meaning of a message by conditionalizing an epistemic state. Analogously, a 'matrix' in quantum theory represents an observation by contracting a particles 'wave function' during the famous 'quantum-leap' [20, 21].

The crucial question that has to be answered in this context is this: How close is the connection between orthomodular lattices and projection lattices on Hilbert spaces (i.e. vector spaces with a defined inner product which capture similarity relations)? All projection lattices are orthomodular lattices. Unfortunately, the converse is not true: not each orthomodular lattice can be represented by a corresponding projection lattice. One needs some additional conditions in order to prove the corresponding representation theorem [40, 30, 36] In a series of papers, Alexandru Baltag and Sonja Smets (e.g., [6, 7]) show the close relationship between a Piron-style lattice-theoretic framework and an explicitly dynamic semantics.

5 Our last issue concerns the marriage of geometric models of meaning with the logical tradition of formal semantics. In cognitive science, geometric models have played a very important role since the beginning. Newer developments are the postulation of (non-propositional) *image-schemes* [33, 34], variants of connectionist models (e.g. [39, 9], conceptual spaces [19] and vector-based computational linguistics (e.g. [35, 38, 31]. It was only recently that the logical branch and the cognitive branch of this development were led together into a new research field called 'quantum cognition' [1, 3, 4, 17, 14, 15, 2].

This brings us to the Erlangen program of Felix Klein (cf. [32]). Most of the achievements of modern theoretical physics are either directly or indirectly related to this program, which seeks to connect geometrical research in mathematics with theoretical physics. Special relativity, general relativity, and quantum mechanics provided beautiful illustrations of the power of Klein's approach. In connection with the concept of symmetry, group theory was seen as a natural way of organizing geometrical knowledge. In connection with Bohr's and Heisenberg's quantum approach it was concluded that "the propositional calculus of quantum mechanics has the same structure as an abstract projective geometry" [8].

The structural similarities between quantum physics and the cognitive realm are a consequence of the *dynamic* and *geometric conception* that underlies both fields. Formal semantics and formal pragmatics will arrive in the 21th century if they follow a research program similar to the Erlangen program in physics – striving to catch the intrinsic nature of natural language interpretation in terms of abstract geometrical structures.

[1] By considering data semantics, Veltman [47] moved away from the idealization of Boolean propositions — however, it is still an idealization not an abstraction yet, in our opinion.

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