Partitions representing change homogeneously

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1 Introduction

As leading figures in Amsterdam's formal semantics scene, Jeroen, Martin and Frank have inspired many semanticists over the years to follow them or in some way respond to their ideas. I am grateful to be somewhere in that non-exclusive disjunction, and wish them the best on their upcoming retirements. The remainder of this note describes some variations on themes familiar from their work, set in the temporal domain. The basic point is to represent change in an interval by partitioning it into subintervals, assuming change is manifested through temporal propositions that are interpreted over intervals.

Fix a set Φ of temporal propositions, hereafter called *fluents*. A Φ-*timeline* $\langle T, \prec, v\rangle$ consists of

(i) a non-empty set T (of "instants" or "moments")

(ii) a linear order $\prec$ on T with the set

$$Ivl(\prec) \;=\; \{I \subseteq T \mid (\forall t, t' \in I)\{t'' \in T \mid t \prec t'' \prec t'\} \subseteq I\}$$

of intervals, and

(iii) a relation $v \subseteq \Phi \times Ivl(\prec)$ consisting of pairs (φ, I) of fluents $\varphi \in \Phi$ and intervals $I \in Ivl(\prec)$ that "v-satisfy" φ.

The equivalence $\approx^v_\varphi$ on the set $Ivl(\prec)$ of intervals I, I' given by

$$I \approx^v_\varphi I' \iff (v(\varphi, I) \iff v(\varphi, I'))$$

induces the partition on $Ivl(\prec)$ that we might, following [GS84], interpret as the question $?\varphi$. There is a coarseness about $\approx^v_\varphi$, however, in equating two intervals I_1 and I_2, neither of which v-satisfies φ, even if *no* subinterval of I_1 v-satisfies φ whereas some subinterval of I_2 does. In this case, I_1 is (φ, v)-homogeneous, while I_2 (which buries φ) is not. More precisely, an interval is (φ, v)-*homogeneous* if it is $\approx^v_\varphi$-equivalent to each of its subintervals

$$I \text{ is } (\varphi, v)\text{-homogeneous} \iff (\forall J \sqsubseteq I)\; I \approx^v_\varphi J$$

where the subinterval relation $\sqsubseteq$ is $\subseteq$ restricted to $Ivl(\prec)$. Writing $I \sqcup I'$ for the smallest interval containing $I \cup I'$, let us refine $\approx^v_\varphi$ to the relation $=^v_\varphi$ given by

$$I =^v_\varphi I' \iff I \sqcup I' \text{ is } (\varphi, v)\text{-homogeneous.}$$

Clearly, $=^v_\varphi$ is contained in $\approx^v_\varphi$, symmetric, and reflexive on (φ, v)-homogeneous intervals. To conclude that $=^v_\varphi$ is an equivalence relation on (φ, v)-homogeneous intervals, it remains to establish transitivity

$$I =^v_\varphi I' \text{ and } I' =^v_\varphi I'' \implies I =^v_\varphi I''$$

for which it is useful to assume

(A1) $\quad v(\varphi, I \cup I') \iff v(\varphi, I)$ and $v(\varphi, I') \quad$ whenever $I, I', I \cup I' \in Ivl(\prec)$.

One half of (A1), $\Longrightarrow$, ensures φ has the so-called subinterval property

$$v(\varphi, I) \implies v(\varphi, J) \quad \text{whenever } J \sqsubseteq I$$

commonly assumed of fluents φ representing statives since [BP72], as is the second half of (A1)

$$v(\varphi, I) \text{ and } v(\varphi, I') \implies v(\varphi, I \cup I') \quad \text{whenever } I \cup I' \in Ivl(\prec).$$

Note that if φ satisfies (A1), then so does $\neg\varphi$ with

$$v(\neg\varphi, I) \iff (\forall J \sqsubseteq I) \text{ not } v(\varphi, J)$$

for all $I \in Ivl(\prec)$.[1] For (φ, v)-homogeneous I, $v(\neg\varphi, I)$ reduces to not $v(\varphi, I)$. Returning to $=^v_\varphi$, we have

Proposition 1. *Assuming (A1), $=^v_\varphi$ is an equivalence relation on (φ, v)-homogeneous intervals.*

In section 2, we extend $=^v_\varphi$ conservatively from (φ, v)-homogeneous intervals to all of $Ivl(\prec)$, including those left out by $=^v_\varphi$, introducing a second assumption that bounds variation. Section 3 shows how to step from a single fluent φ satisfying (A1) to a set X of such, with an interval defined to be (X, v)-*homogeneous* if it is (φ, v)-homogeneous, for every $\varphi \in X$. Finally, section 4 restricts X to be finite, passing over to strings. We assume throughout that (A1) holds for every fluent φ.

2 Partitions based on homogeneous intervals

For any set $\mathbb{J} \subseteq Ivl(\prec)$ of intervals, let $f_\mathbb{J} : Ivl(\prec) \to 2^\mathbb{J}$ be the function mapping an interval to the set of intervals in $\mathbb{J}$ that intersects with it

$$f_\mathbb{J}(I) \;=\; \{J \in \mathbb{J} \mid J \cap I \neq \emptyset\} \;.$$

Equating intervals I and I' that intersect the same intervals in $\mathbb{J}$, we get an equivalence relation $\approx_\mathbb{J}$ on $Ivl(\prec)$ from the clause

$$I \approx_\mathbb{J} I' \iff f_\mathbb{J}(I) = f_\mathbb{J}(I').$$

In practice, we choose $\mathbb{J}$ to be a partition of T. But to relate $\approx_\mathbb{J}$ to $\approx^v_\varphi$, we need to constrain $\mathbb{J}$ further.

Proposition 2. *If $\mathbb{J} \subseteq Ivl(\prec)$ is a partition of T, then*

$$\approx_\mathbb{J} \text{ refines } \approx^v_\varphi \iff \text{each } J \in \mathbb{J} \text{ is } (\varphi, v)\text{-homogeneous.}$$

Proof: For $\Longrightarrow$, observe that if $J \in \mathbb{J}$ were not (φ, v)-homogeneous, then J would have two subintervals that are in $\approx_\mathbb{J}$ but not in $\approx^v_\varphi$. To prove the converse, note that for every interval I, there are at most two intervals $J \in f_\mathbb{J}(I)$ such that *not* $J \subseteq I$ (as a third interval would puncture a hole through I). That is,

$$\bigcup f_\mathbb{J}(I) \;=\; I \cup J_1 \cup J_2 \qquad \text{for some } J_1, J_2 \in f_\mathbb{J}(I).$$

[1] In [AF94], $\neg$ is called *strong negation* (page 540), while in [Ham71], it is *predicate negation* (page 131).

Thus, if $v(\varphi, I)$, then $v(\varphi, \bigcup f_{\mathbb{J}}(I))$, as φ satisfies (A1) and each $J \in \mathbb{J}$ is (φ, v)-homogeneous. And again, $v(\varphi, I')$ for any $I' \approx_{\mathbb{J}} I$. □

Clearly, the bigger the (φ, v)-homogeneous intervals in $\mathbb{J}$, the coarser $\approx_{\mathbb{J}}$ is. The remainder of this section considers the special case where $\mathbb{J}$ is finite and consists of the largest (φ, v)-homogeneous intervals. We lift $\prec$ to intervals I, I' universally for *whole* precedence $\prec$

$$I \prec I' \iff (\forall t \in I)(\forall t' \in I')\ t \prec t',$$

and define a (φ, v)-*alternation* to be a finite sequence $J_1 J_2 \ldots J_n$ of intervals $J_i \in \mathit{Ivl}(\prec)$ such that whenever $1 \leq i < n$, $J_i \prec J_{i+1}$ and

$$v(\varphi, J_{i+i}) \iff \text{not } v(\varphi, J_i).$$

A (φ, v)-*scale* is a (φ, v)-alternation $J_1 \ldots J_n$ such that $\bigcup_{i=1}^{n} J_i = T$ and each J_i is (φ, v)-homogeneous. An obvious property of (φ, v)-scales is uniqueness.

Proposition 3. *For all φ and v (satisfying (A1)), there is at most one (φ, v)-scale.*

The next question is existence. Let us agree to call a (φ, v)-alternation $J_1 \ldots J_n$ *long* if its length n is greater than or equal to the length of every (φ, v)-alternation. Clearly, a (φ, v)-scale is long. Conversely, we can turn any long (φ, v)-alternation $\mathbf{J} = J_1 \ldots J_n$ into a (φ, v)-scale $s(\mathbf{J}) = J'_1 \ldots J'_n$ as follows. For $n = 1$, $J'_1 = T$. Otherwise, $n > 1$ and

$$\begin{aligned} J'_1 &= \{t \in T \mid \{t\} \prec J_2 \text{ and } \{t\} \approx^v_\varphi J_1\} \\ J'_n &= \{t \in T \mid J_{n-1} \prec \{t\} \text{ and } \{t\} \approx^v_\varphi J_n\} \end{aligned}$$

and for $1 < i < n$,

$$J'_i = \{t \in T \mid J_{i-1} \prec \{t\} \prec J_{i+1} \text{ and } \{t\} \approx^v_\varphi J_i\}.$$

Before leaping to the conclusion that $s(\mathbf{J})$ is a (φ, v)-scale, we pause for an instructive example. Let T be the set $\mathbb{N} = \{0, 1, 2, \ldots\}$ of non-negative integers (under the usual ordering) and $\hat{\varphi}$ pick out intervals $I \in \mathit{Ivl}(\mathbb{N})$ that are bounded

$$v(\hat{\varphi}, I) \iff (\exists n \in \mathbb{N})(\forall t \in I)\ t < n\ .$$

A long $(\hat{\varphi}, v)$-alternation has length 2, but its last subinterval cannot be $(\hat{\varphi}, v)$-homogeneous. To sidestep pesky fluents such as $\hat{\varphi}$, let us strengthen (A1) and define φ to be *v-pointwise* if for every interval I,

$$v(\varphi, I) \iff (\forall t \in I)\ v(\varphi, \{t\}).$$

Proposition 4. *If φ is v-pointwise, and a (φ, v)-alternation $\mathbf{J}$ is long, then $s(\mathbf{J})$ is a (φ, v)-scale.*

The existence of a long (φ, v)-alternation (or equivalently, of a (φ, v)-scale) can be put as

(A2) there is an integer k such that every (φ, v)-alternation has length $< k$.

In effect, (A2) says that as an instrument for observing T (or better, $\prec$), φ has a limited shelf life and resolution. To overcome these limitations, we might work with not just one fluent φ but many. Of course, many fluents φ can be useful, whether or not they satisfy (A2). That said, we might get around a fluent violating (A2) by breaking it down into infinitely many fluents, each satisfying (A2).

3 From one fluent to many

Recall that the *(dependent) product of* a family $\{A_x\}_{x\in X}$ of sets A_x indexed by X is the set

$$\prod_{x\in X} A_x \;=\; \{f : X \to \bigcup_{x\in X} A_x \mid (\forall x \in X)\ f(x) \in A_x\}$$

of functions f with domain X such that $f(x) \in A_x$ for every $x \in X$.

Proposition 5. *Let X be a set of fluents φ satisfying (A1) and $\{\mathbb{J}(\varphi)\}_{\varphi\in X}$ be a family of partitions $\mathbb{J}(\varphi)$ of T into (φ, v)-homogeneous intervals. The intersection $\bigcap_{\varphi\in X} \approx_{\mathbb{J}(\varphi)}$ is $\approx_{\mathbb{J}(X)}$, where $\mathbb{J}(X)$ is a partition of T into (X, v)-homogeneous intervals given by*

$$\mathbb{J}(X) \;=\; \{\bigcap_{\varphi\in X} f(\varphi) \mid f \in \prod_{\varphi\in X} \mathbb{J}(\varphi)\} - \{\emptyset\}.$$

From finite partitions $\mathbb{J}(\varphi)$ (e.g. (φ, v)-scales, assuming (A2)), Proposition 5 may produce an infinite partition $\mathbb{J}(X)$, provided X is infinite. But even if X is finite, the construction holds some interest.

4 From partitions to strings

When X is finite, and each $\mathbb{J}(\varphi)$ in Proposition 5 is finite, $\mathbb{J}(X)$ reduces to a finite set $\{J_1, \ldots, J_n\}$ of (X, v)-homogeneous intervals $J_1 \prec \cdots \prec J_n$, with

$$I \approx_{\{J_1\ldots J_n\}} I' \iff (\forall i \in [1, n])\ (J_i \cap I \neq \emptyset \iff J_i \cap I' \neq \emptyset).$$

(where $[j, k]$ is the set of integers $\geq j$ and $\leq k$). The X-timeline $\langle [1, n], <, \hat{v}\rangle$ given by

$$\hat{v}(\varphi, [j, k]) \iff v(\varphi, \bigcup_{i=j}^{k} J_i) \qquad \text{whenever } \varphi \in X \text{ and } 1 \leq j \leq k \leq n$$

is the image of $\langle T, \prec, v\rangle$ under the homomorphism $f : T \to [1, n]$ mapping $t \in T$ to the unique $i \in [1, n]$ such that $t \in J_i$. Note

(i) for every $I \in \mathit{Ivl}(\prec)$, $\{f(t)|t \in I\} = [j, k]$ for some j, k such that $1 \leq j \leq k \leq n$ and

$$v(\varphi, I) \iff \hat{v}(\varphi, [j, k]) \qquad \text{for every } \varphi \in X$$

(ii) for all $I, I' \in \mathit{Ivl}(\prec)$,

$$I \prec I' \iff (\forall t \in I)(\forall t' \in I')\ f(t) < f(t')$$

and for every $\varphi \in X$,

$$I \approx^{v}_{\varphi} I' \iff \{f(t)|t \in I\} \approx^{\hat{v}}_{\varphi} \{f(t')|t' \in I'\}.$$

Were we not interested in how T maps onto $[1, n]$, we could reduce the sequence $J_1 \ldots J_n$ further to a string $\alpha_1 \cdots \alpha_n$ with

$$\alpha_i \;=\; \{\varphi \in X \mid v(\varphi, J_i)\} \qquad \text{for } 1 \leq i \leq n$$

so that

$$\hat{v}(\varphi, [j,k]) \iff \varphi \in \bigcap_{i=j}^{k} \alpha_i \qquad \text{whenever } \varphi \in X \text{ and } 1 \leq j \leq k \leq n.$$

These strings have a role to play in the semantics of tense and aspect, tracking X-changes, of which events at granularity X are part. Or so I argue, most recently in [Fer13]. On this point, I close, as I began, with a personal note. At an early stage of my work representing events through strings, Frank provided encouragement, which has somehow seen me through three or four Amsterdam Colloquium talks on the subject (not to mention a few hiccups along the way). Whether or not I have made more of this kindness than you meant, Frank, thank you.

References

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