

# Applications of IF-logic

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## 1 Introduction

The ILLC is an interdisciplinary institute, in which several fields are connected, and bridges are laid between the fields. Some examples are the following. The discovery by the computer scientist Peter van Emde Boas of recursive structures in the work of Jeroen Groenendijk and Martin Stokhof on information about information. The influence of philosopher (and co-promotor) Renate Bartsch on the dissertation of the mathematician Theo Janssen on Montague grammar. The important role of the philosopher Frank Veltman on the dissertation by the mathematician Joost Joosten on the foundations of arithmetic. The stimulating guidance of Johan van Benthem on many philosophical dissertations. And recently the connection between the work of the mathematical logician Väänänen and inquisitive semantics by the philosopher Jeroen Groenendijk.

In this contribution a topic will be presented exemplifying such bridges: Independence Friendly Logic. Its applications in mathematics (continuity), philosophy (de dicto - de re), and computer science (Skolem forms) will be discussed; there also are applications in linguistics (branching quantifiers sentences) and in logic (Henkin quantifiers). In the appendix it will be sketched how the ground was prepared for this interdisciplinary institute in Amsterdam.

## 2 IF logic

In predicate logic quantifiers may depend on the quantifiers in whose scope they occur: scope indicates possible dependencies. It is, however, *not* possible in predicate logic to express that a quantifier has to be independent of another one. Independence friendly logic is a generalization of predicate logic in which this is possible. Independence is indicated by adding a  $/x$  as subscript to the variable mentioned with the quantifier, an example is  $\forall x \exists y_{/x} \psi$  where  $y$  should be independent from the value of  $x$ . Below it will be explained how this is formalized. Independence friendly logic, henceforth IF logic (or IF), is introduced by J. Hintikka, and advocated in a number of publications; the main ones are Hintikka (1996), and Hintikka & Sandu (1997).

The interpretation of a sentence  $\varphi$  from IF logic proceeds as a game between 2 players,  $\forall$ belard and  $\exists$ loise:

- $\exists$ loise tries to confirm  $\varphi$ . She chooses a value for each  $\exists y$  and  $\exists y_{/x}$ , and a disjunct for each  $\vee$ . The sentence is called *true* if she has a winning strategy (a notion to be clarified below).

- $\forall$ belard tries to refute  $\varphi$ . He chooses a value for each  $\forall x$  and a conjunct for each  $\wedge$ . The sentence is called *false* if he has a winning strategy.

Negation (which causes a role switch) is not considered in this paper. Originally (in)dependence also involved the connective  $\vee$ , that option is not considered here. Of course, the game is generalized to formulas with free variables.

As introduction we consider two examples. The first is:  $\forall x \exists y [x = y]$ . Eloise has winning strategy: she chooses the same value as  $\forall$ belard has chosen before. Therefore this sentence is *true*. The second is:  $\forall x \exists y_{/x} [x = y]$ . Now Eloise has *not* the option to use the value chosen before by  $\forall$ belard. By luck she may win, but she has no strategy that guarantees her to win. Therefore the sentence is not *true*. Also  $\forall$ belard winning strategy: hence the sentence is not *false* either. In the sequel we will only be interested in the issue whether a sentence is *true*.

Next we consider two other (informative) examples. The first one is:

$$\forall x \exists y \forall z \exists w [y > x \wedge w > x + y + z].$$

Due to the scope of the quantifiers  $w$  depends on  $x$ ,  $y$ , and  $z$ . Since the value of  $x$  is available for the calculation by Eloise of the value of  $w$ , the value of  $y$  is not needed: that value can be recalculated using the value of  $x$  (using the same strategy as she used before). So we might equivalently say:  $w$  depends on  $x$  and  $z$ .

The second example is:

$$\forall x \exists y \forall z \exists w_{/x} [y > x \wedge w > z + x].$$

Here it is indicated that the  $w$  should be independent of  $x$ . But in order to make sense of this requirement, the value of  $y$  (which gives information on  $x$ ) should not be used:  $w$  should not be defined to be equal to  $z + y$ . Obeying this restriction, the sentence is not *true*.

The last examples motivate the general principle that existential quantifiers do not depend on the values of other existential quantifiers: either the required values can be recalculated, or the first existential quantifier uses a value that should not be used by the second one.

This explains the following description of a strategy:

- For  $\exists y$  and  $\vee$ . A strategy is a function with as arguments: previous choices of the *opponent* for values of variables. This function yields respectively a value, or a  $L, R$  decision. Likewise the strategies for  $\forall x$  and  $\wedge$  can be described.
- For  $\exists y_{/x}$ . A strategy is a function with as arguments: previous choices of of the *opponent* for values of variables except for his choice for  $x$ .

So in fact the strategies for  $\exists x$  are, for classical predicate logic, the traditional Skolem functions; in case of independence, some arguments are omitted.

### 3 Mathematics: (uniform) continuity

Immediate examples in mathematics of independence are the definitions of *continuity* and *uniform continuity* (Hintikka 1996). Intuitively speaking, a function is called continuous if it can be drawn as one curve; so without having to lift the pencil. An example is  $f(x) = x^2$ . An example of a non-continuous function is the one defined by  $f(x) = -1$  for  $x < 0$  and  $f(x) = 1$  for  $x \geq 0$  (there is a jump for  $x = 0$ ). Also  $f(x) = \frac{2x}{x}$  is not continuous because it is undefined in 0. The notion of continuity is formalized by stating that if  $y$  is chosen close enough to  $x$ , then  $f(y)$  is as close to  $f(x)$  as required. In other words, if it is asked that  $f(y)$  and  $f(x)$  differ less than the small number  $\varepsilon$ , then a small number  $\delta$  can be found that guarantees this effect for  $y$  and  $x$  differing less than  $\delta$ . This phrase is represented in predicate logic by the well known  $\varepsilon$ - $\delta$  definition:

$$(1) \text{ Continuity: } \forall \varepsilon \forall x \exists \delta \forall y [ |x - y| < \delta \rightarrow |f(x) - f(y)| < \varepsilon ]$$

In some cases a given value for  $\varepsilon$  suffices to find a  $\delta$ , because then the  $\delta$  does not depend on  $x$ . This is, for instance, the case if the function has everywhere the same relation between the distance of the arguments and the distance between the function values. An example is  $f(x) = 2x + 2$  (because it is drawn as a straight line). Another situation arises if the function has a steepest part: then the relation required for the steepest part can be used everywhere. An example is  $f(x) = \sin(x)$ , because this function has its steepest part for  $x = 0$ . In such cases the function is called *uniformly continuous*. This is expressed in predicate logic by:

$$(2) \text{ Uniformly continuous: } \forall \varepsilon \exists \delta \forall x \forall y [ |x - y| < \delta \rightarrow |f(x) - f(y)| < \varepsilon ]$$

One sees that replacement of the word *continuous* by *uniformly continuous* results in a change in the quantifier structure of the logical sentence.

Such a change of structure is not attractive; one would prefer that a local change in the sentence would correspond with a local change in the logical representation. In a compositional analysis of the sentences this would even be required. In IF logic such an analysis is possible. Then replacing  $\exists \delta$  by  $\exists \delta/x$  yields the desired result:

$$(3) \text{ Uniformly continuous (IF): } \forall \varepsilon \forall x \exists \delta/x \forall y [ |x - y| < \delta \rightarrow |f(x) - f(y)| < \varepsilon ].$$

The above example is presented by Hintikka (1996, p. 9). He also suggests to use it for comparable notions like (*uniformly*) *differentiable* (Hintikka 1996, p. 74). Hodges (1997, p. 51) presents another type of example. It is from algebraic geometry (Lang 1962, p. 67):

Let  $V$  be a non-singular projective variety, and  $X$  a hyperplane section. Given an integer  $d$ , there exists a positive integer  $e$  depending only on  $V$  and  $d$  such that for any positive divisor  $Y$  on  $V$  of degree  $d$ , the divisors  $Y + eX$  and  $-Y + eX$  are ample.

A standard first-order symbolization would be:  $\forall V \forall X \forall d \exists e \forall Y \varphi(V, d, e, X, Y)$ . But here  $X$  depends on  $e$ , and that explicitly is not intended. In IF logic this can easily be expressed:  $\forall V \forall X \forall d \exists e_{/X} \forall Y \varphi(V, d, e, X, Y)$ .

I would like to draw attention to an advantage of such a compositional analysis. Representations can be used in all contexts, for instance to express that *all functions  $g$  from set  $G$  are uniformly continuous*. All that has to be said is that  $\delta$  is independent of  $x$ , whereas its dependence on  $g$  follows from the quantifier structure.

(4) Uniformly Continuous for a set:

$$\forall g \in G \forall \varepsilon \forall x \exists \delta_{/x} \forall y [ |x - y| < \delta \rightarrow |f(x) - f(y)| < \varepsilon ]$$

There is an alternative approach for IF logic, called Dependence Logic (Väänänen 2007). It follows the opposite approach, and indicates for a new variable explicitly on which variables it depends at most. So for *uniformly continuous* it says on which variables  $\delta$  depends at most. In Janssen (2012) it is shown that for principled reasons DL cannot provide a formalization for *uniformly continuous* that may be used in a compositional way: if the sentence is extended with additional factors, incorrect results are obtained.

## 4 Philosophy: de dicto – de re

The sentence

(5) *Mary believes that a stranger crippled John's cow*

is ambiguous. In the one reading, called *de dicto* reading, the sentence says that Mary does not have a particular person in mind, but that she believes that whatever precisely happened, it must be some stranger who crippled John's cow. In the other reading, the *de re* reading, the sentence says that there is a particular person of whom she believes that he crippled John's cow (for instance a stranger Mary saw last night). The cow is in all readings assumed to be a unique real cow. The *de dicto - de re* ambiguity has a long history in philosophy (McKay & Nelson 2011), but that will not be considered here.

Hintikka's claim that IF logic is suitable for the *de dicto - de re* ambiguity is not worked out; the formalization in this paper is my own. Tense will not be considered because that is not relevant for our problem.

The meaning of a sentence will be formalized as the set of possible worlds in which the sentence is true. Explicit variables for possible worlds will occur in the predicates, typically  $w$  and  $v$  will be used. For instance, the meaning of *John loves Suzy*, viz.  $Love(w, J, S)$ , will hold for some values of  $w$ , but not for other values.

The verb *believe* will get a modal interpretation (this type of interpretation is introduced by Hintikka (2005), and is standard in epistemic logic (van Ditmarsch, van der Hoek & Kooi 2007)). The sentence *Mary believes that John loves Suzy* is understood

as that *John loves Suzy* is true in the worlds that are compatible with Mary's beliefs. That is not a unique world because Mary may not have an opinion concerning Suzy loving John, or not concerning Goldbach's conjecture. The set of worlds compatible with her beliefs in  $v$  is denoted  $Bel(v, M)$ , and is called the 'set of belief alternatives of Mary' (in  $v$ ). The meaning with respect to  $v$  of *Mary believes that John loves Suzy* is denoted by  $\forall w \in Bel(v, M) Love(w, J, S)$ . An alternative formulation would be  $\forall w [w \in Bel(v, M) \rightarrow Love(w, J, S)]$ , and still another variant is used by Montague (1973) where the meaning of *believe* is expressed as a relation between Mary and the set of worlds in which  $Love(w, J, S)$  holds.

Hence the meaning representations in IF for the two readings of (5) are:

$$(6) \text{ de re (IF): } \forall w \in Bel(v, M) \exists x_{/w} [Str(w, x) \wedge \exists y_{/w} [Cow(w, y, J) \wedge Cr(w, x, y)]]$$

$$(7) \text{ de dicto (IF): } \forall w \in Bel(v, M) \exists x [Str(w, x) \wedge \exists y_{/w} [Cow(w, y, J) \wedge Cr(w, x, y)]]$$

The variable  $v$  gets as value the world with respect to which we interpret the formula. In (6) the  $\exists x_{/w}$  says that the  $x$  is chosen independently of  $w$ , so it is some particular person (the *de re* reading), whereas in (7) the  $_{/w}$  does not occur with  $\exists x$ , so  $x$  may depend on  $w$  (the *de dicto* reading).

Next the question is how to obtain them in a compositional way. Following traditional syntax, we take as structure for the sentence:

$$(8) [Mary [believes that [[a stranger] [crippled [John's cow]]]]]$$

The smallest parts are not indicated, for instance also  $a$  and *stranger* are parts.

We start with the meaning of the smallest subsentence: *a stranger crippled John's cow*. Of course, its meaning has to be formed from the meanings of the words from which it is built, but this technique is standard (see an introduction like Dowty, Wall & Peters (1981) or Gamut (1991)). Roughly speaking, a determiner denotes a generalized quantifier and a noun denotes a set of entities. The representation of the *de re* reading of the determiner  $a$  is, in an extension of IF with lambda's,  $\lambda P \lambda Q [\exists x_{/w} [P(w, x) \wedge Q(w, x)]]$ . For the *de dicto* reading  $\exists x$  is used.

First we give the meaning representation for the *de re* reading of (9):

$$(9) \text{ A stranger crippled John's cow.}$$

$$(10) \text{ de re (IF): } \exists x_{/w} [Str(w, x) \wedge \exists y_{/w} [Cow(w, y, J) \wedge Cr(w, x, y)]]$$

Notice that (10) contains a free variable  $w$  denoting the world with respect to which we interpret this sentence. If the sentence occurs as main sentence, that will be the actual world, but embedded after *believe* it can be a belief alternative.

The next step is to embed (9) under *believe*, yielding (11)

$$(11) \text{ believe that a stranger crippled John's cow.}$$

$$(12) \text{ de re (IF): } \forall w \in Bel(v, p) \exists x_{/w} [Str(w, x) \wedge \exists y_{/w} [Cow(w, y, J) \wedge Cr(w, x, y)]]$$

Now the variable  $w$  is bound by  $\forall w$ , which means that all worlds are considered that are possible according to the belief alternatives in world  $v$  of person  $p$ , where  $v$  and  $p$  are variables.

Next we combine (11) with *Mary*, yielding (13). On the semantic side the meaning of *Mary*, being  $\lambda P(M)$  is applied to (12). The result is equivalent with (14).

(13) *Mary believes that a stranger crippled John's cow.*

(14) de re (IF):  $\forall w_{\in Bel(v,M)} \exists x_{/w} [Str(w, x) \wedge \exists y_{/w} [C(w, y, J) \wedge Cr(w, x, y)]]$

Thus the meaning representation we aimed at, is obtained in a compositional way.

An advantage of the compositional approach is that the same parts can be used for the production of other sentences. For instance, we may combine *believes that a stranger crippled John's cow* as well with *every woman*, yielding:

(15) *Every woman believes that a stranger crippled John's cow.*

This sentence has the same ambiguity as we have discussed above: the stranger may either depend on the belief alternatives of the woman under consideration (the *de dicto* reading), or be independent of those alternatives (the *de re* reading), but still dependent on which woman is considered. There also is a reading in which all women suspect the same individual, but that reading will not be considered here.

We obtain the *de re* the reading of (15) in the same way as the *de re* the reading of (13). The final result will not be surprising; it is equivalent with:

(16) de re (IF):

$\forall z [W(v, z) \rightarrow \forall w_{\in Bel(v,z)} \exists x_{/w} [Str(w, x) \wedge \exists y_{/w} [Cow(w, y, J) \wedge Cr(w, x, y)]]]$

This shows that in IF logic the *de re* reading of (15) can be obtained in a compositional way.

The problem can easily be generalized. We might add another factor which has influence on the choice of the stranger. For instance:

(17) *In every town every woman believes that a stranger crippled John's cow.*

We may use the meaning of (15) and embed that under the scope of the quantifier for *every town*.

Further generalizations are possible. Suppose that in a sentence the existence of an individual (or entity) is stated. Then several factors (place, time, person, someone's belief alternatives, ...) may have influence on which individual is intended. In most sentences these factors are not mentioned explicitly, and assumed to be fixed and given by context. But in case a factor is mentioned explicitly and happens to be universally quantified (e.g. by *every woman* or *in every town*), the different values of this factor determine different individuals. Compositionality requires that the meaning for the

expression without the explicit quantification can also be used when the factor is universally quantified and thereby becomes dependent of that factor. We have seen that IF logic has this property.

In the previous section we mentioned Dependence Logic. In Janssen (2012) it is argued that DL that does not work for this phenomenon: although the *de dicto* and the *de re* reading can be expressed, they cannot be used in a compositional way.

## 5 Computer Science: Theorem Proving

An important proof method in logic is *resolution*, it occurs in many introductions to logic, and it forms the foundation for the programming language *Prolog*. As a preliminary step *Skolem forms* have to be formed. The standard method for obtaining them is with simple local transformations. The result is, however, not unique, and is often too complicated. This causes problems for theorem proving, and makes them less attractive for teaching. We will illustrate these problems, and show that using IF logic eliminates these problems.

The textbook method to obtain Skolem forms has the following steps:

1. The formula is rewritten such that the only connectives are  $\wedge$ ,  $\vee$  and  $\neg$ , where the negation only occurs for basic formulas. Furthermore bound variables are renamed such that each quantifier binds its own variable.
2. Put the formula in prenex normal form. This means that the formula is brought in an equivalent form that starts with a block of quantifiers that have scope over a quantifier free formula. The rules that are needed are:
  - (a) if  $x$  does not occur in  $\psi$ :  $\forall x[\varphi] \wedge \psi \equiv \forall x[\varphi \wedge \psi]$ ,  $\exists x[\varphi] \vee \psi \equiv \exists x[\varphi \vee \psi]$
  - (b)  $y$  does not occur in  $\varphi$ :  $\varphi \wedge \forall y[\psi] \equiv \forall y[\varphi \wedge \psi]$ ,  $\varphi \vee \exists x\psi \equiv \exists x[\varphi \vee \psi]$
3. Skolemize. This means that for each  $\exists$  quantifier a fresh function is introduced that has as arguments the values of variables that are bound by universal quantifiers that have scope over that existential quantifier. If there are no such quantifiers, then a fresh constant is introduced. These functions are called Skolem functions, and these constant Skolem constants. So the remaining quantifiers all are universal ones, these usually are not represented explicitly.

Using a single example (with a disjunction), two strategies for obtaining Skolem functions are illustrated:

1. Give the leftmost quantifiers wide scope first, thereafter the other quantifiers.
2. Start with the rightmost quantifiers.

The main steps of the first strategy are:

1.  $\forall x\exists y\forall z R(x, y, z) \vee \exists u\forall v\exists w Q(u, v, w)$ . The initial sentence.
2.  $\forall x[\exists y\forall z R(x, y, z) \vee \exists u\forall v\exists w Q(u, v, w)]$ . So  $\forall x$  has obtained wide scope over the formula.

3.  $\forall x \exists y \forall z [R(x, y, z) \vee \exists u \forall v \exists w Q(u, v, w)]$ . The other quantifiers from the leftmost subformula have obtained wide scope over the formula.
4.  $\forall x \exists y \forall z \exists u [R(x, y, z) \vee \forall v \exists w Q(u, v, w)]$ . The first formula in from the other subformula has wide scope.
5.  $\forall x \exists y \forall z \exists u \forall v \exists w [R(x, y, z) \vee Q(u, v, w)]$ . The other quantifiers from the rightmost subformula have obtained wide scope over the formula.
6. Skolemise:  $\forall x \forall z \forall v [R(x, f(x), z) \vee Q(g(x, z), v, h(x, z, v))]$

This result is too complicated. In the original formula the  $\exists u$  is not in the scope of any quantifier, so it should not depend on  $\forall x$ ; a fresh constant would be sufficient. And the  $\exists w$  is only in the scope of  $\forall v$ , so should not depend on  $\forall x$  and  $\forall z$ .

The second strategy is to give the rightmost quantifiers first wide scope:

1.  $\forall x \exists y \forall z R(x, y, z) \vee \exists u \forall v \exists w Q(u, v, w)$ . The initial sentence.
2.  $\exists u [\forall x \exists y \forall z R(x, y, z) \vee \forall v \exists w Q(u, v, w)]$ . So  $\exists u$  from the rightmost subformula has obtained wide scope over the formula.
3.  $\exists u \forall v \exists w [\forall x \exists y \forall z R(x, y, z) \vee Q(u, v, w)]$ . The other quantifiers from the rightmost subformula have obtained wide scope over the formula.
4.  $\exists u \forall v \exists w \forall x \exists y \forall z [R(x, y, z) \vee Q(u, v, w)]$ . All quantifiers from the leftmost subformula have obtained wide scope over the formula.
5. Skolemise:  $\forall u \forall x \forall z [R(x, f(v, x), z) \vee Q(a, v, g(v))]$ . The initial quantifier  $\exists u$  has introduced of a (fresh) constant  $a$ .

Also this result is too complicated: the  $y$  (i.e.  $f(v, x)$ ) has nothing to do with the  $\forall v$ .

Each order of extraction of quantifiers is allowed; other results are possible as well. So there is *no* unique Skolem form. Furthermore the results are too complicated. Both aspects are not attractive in theorem proving and certainly not in a teaching situation. There are alternative methods in the literature for obtaining Skolem forms, but these are more complicated, and also their result is not always the simplest possible. We will show that using IF logic these problems can be solved.

We use the following definitions and theorem.

**Definitions**  $\psi|_x$  means that  $|_x$  is added to all existential quantifiers in  $\psi$ .

$\varphi \equiv_x \psi$  means that the formulas are truth-equivalent for all sets of assignments that involve a finite set of variables that does not include  $x$ .

**Theorem** If  $x$  does not occur in  $\psi$  nor in the set of variables  $Y$ , then

$$\exists x|_Y [\varphi] \vee \psi \equiv_x \exists x|_Y [\varphi \vee \psi], \quad \forall x [\varphi] \vee \psi \equiv_x \forall x [\varphi \vee \psi|_x].$$

If  $x$  does not occur in  $\varphi$  nor in  $Y$ , then

$$\varphi \vee \exists x|_Y \psi \equiv_x \exists x|_Y [\varphi \vee \psi], \quad \varphi \vee \forall x \psi \equiv_x \forall x [\varphi|_x \vee \psi].$$

And analogously for  $\wedge$ .

We now apply these new rules to the previous example. We follow the strategy leftmost first, but (we claim) that the final result does not depend on the order in which we extract the quantifiers, and the result is in some sense the simplest possible.

1.  $\forall x \exists y \forall z R(x, y, z) \vee \exists u \forall v \exists w Q(u, v, w)$ .



2.  $\forall x[\exists y\forall z R(x, y, z) \vee \exists u_{/x}\forall v\exists w_{/x} Q(u, v, w)]$ . The  $\forall x$  has received wide scope, but the  $\exists$ -quantifiers that came in its scope are now marked for independence.
3.  $\forall x\exists y\forall z [R(x, y, z) \vee \exists u_{/xz}\forall v\exists w_{/xz} Q(u, v, w)]$ . The same for the other quantifiers in the leftmost formula.
4.  $\forall x\exists y\forall z\exists u_{/xz}[R(x, y, z) \vee \forall v\exists w_{/xz} Q(u, v, w)]$ . Now for the first quantifier in the rightmost subformula.
5.  $\forall x\exists y\forall z\exists u_{/xz}\forall v\exists w_{/xz}[R(x, y, z) \vee Q(u, v, w)]$ . then for the other quantifiers.
6. Skolemise:  $\forall x\forall z\forall v[R(x, f(x), z) \vee Q(a, v, g(v))]$ . This is simpler than with predicate logic. Note that if an existential quantifier does not depend on universally quantified variable, that variable does not occur as argument in the Skolem function (see also Section 2).

The three examples illustrate:

1. The resulting Skolem forms are simpler than those obtained with the standard method.
2. The method is as simple as standard method. For extracting only local rules are needed.
3. Therefore more attractive for theorem proving (although it does not take away the source of complexity of theorem proving)
4. The method is in any case more attractive for teaching, because the results are unique, and simpler.

Again one might think of DL logic. However also for this application that logic seems not suitable.

## Appendix: Before the ILLC

The story starts long ago, before I was a student at the university. Prof. Beth was a logician with a broad interest, for instance in the new developments in syntax by Chomsky. Many colleagues in Amsterdam were negative about the Chomsky's work; for instance, prof. Reichling, the professor of general linguistics, stated that application of Chomsky's engineering methods to language would demolish its beauty.

When professor Beth had passed away, logicians were hired on a temporarily basis to replace him. Prof. Staal (general philosophy) had a great interest in the new developments in linguistics, and probably thanks to his connections, Richard Montague was in spring 1966 for half a year in Amsterdam. They organized together a course on natural language in which the modern syntactic ideas and Montague's logical ideas were presented. Each of the teachers explained how in his opinion a certain phenomenon should be treated. It was not always clear that they understood each others contribution.

Furthermore, Staal organized on Friday evenings a working group in which the newest developments from the USA were presented. Among the participants were Verkuyl, Seuren, Dik, Kooij and Brandt Corstius. I mention them because they all got a university position, mainly in Amsterdam, and many became later full professor.

Thus they contributed to the spread of the use mathematical techniques for natural language. Remarkably Hans Kamp was not there; he just had received a grant to visit Montague for a year at UCLA (so he missed his visit to Amsterdam).

Some years later Dik became full professor of General Linguistics in Amsterdam. Although he was not a Chomskyan himself, he was broadly minded and organized courses on modern developments such as transformational grammar and generative semantics. I participated in many of these. An import step was made when he organized a course on Montague's 'The proper treatment of Quantification in ordinary English'. Students with different backgrounds attended, I did, and so did Martin Stokhof and Jeroen Groenendijk. Soon we three started a privately organized reading group on formal semantics; also Alice ter Meulen and Jaap Hoepelman participated, and occasionally Paul van Ulsen. Thus a group of students emerged with an interest in formal semantics.

Martin and Jeroen became student members of the committee who had to hire a new professor for philosophy of language. Dik was chairman. They looked for a person with an interest in formal semantics. First they tried to hire Dieter Wunderlich, and when that failed, they found Renate Bartsch. It meant a strengthening of formal semantics in Amsterdam.

Renate organized a crash course on Montague grammar, Dick de Jongh was one of the teachers. Among the participants was Henk Verkuyl. For him the connection with Montague grammar was important because formal semantics still was not appreciated in the Amsterdam Dutch language department. This course was followed the next year by a conference on 'Montague grammar and related topics'. There started my series of more then ten subsequent contributions to this conference. Furthermore Martin, Jeroen and I started a bi-weekly colloquium on formal semantics where people with different background met each other, such as Peter van Emde Boas from the Mathematical Center, Peter Hendriks from Slavic languages in Leiden, and Remko Scha from the Philips laboratories. We became a national platform. Moreover, this cooperation was the kernel from which the later ILLC and its predecessor ITLI was formed.

The most important action came when the department of mathematics in the mid eighties had to shrink from eleven full professors to seven. They had a chair for Foundations of Mathematics (Troelstra), and one for Mathematical Logic (Löb). When Löb retired, they decided that one professor in the field of logic was enough. In that situation Dick de Jongh took action. He organized support from philosophy (Bartsch) and from Computer Science (Herzberger and van Emde Boas). They sent a letter to higher echelons at the university arguing that the applications of logic outside mathematics to philosophy, linguistics and computer science were as alive as ever and to keep up with its developments a special professorship was needed. And it was decided that such a position had to be created.

A committee was formed, consisting in Troelstra, de Jongh, Bartsch, van Emde Boas, H. Lenstra and student Jaap van Oosten. It was not an easy task with members with such divergent interests. The result was that they decided to appoint Johan van

Benthem. He turned out to be an important and powerful person for the development of the field. A mathematician once said: we almost succeeded, logic was almost dead in Amsterdam, and then Johan came. Indeed, soon the ITLI was formed (‘Instituut voor taal, logica en informatie’), which later got its English name ILLC (‘Institute for Logic, Language and Information’).

I hope that this institute can maintain for long its interdisciplinary profile.

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