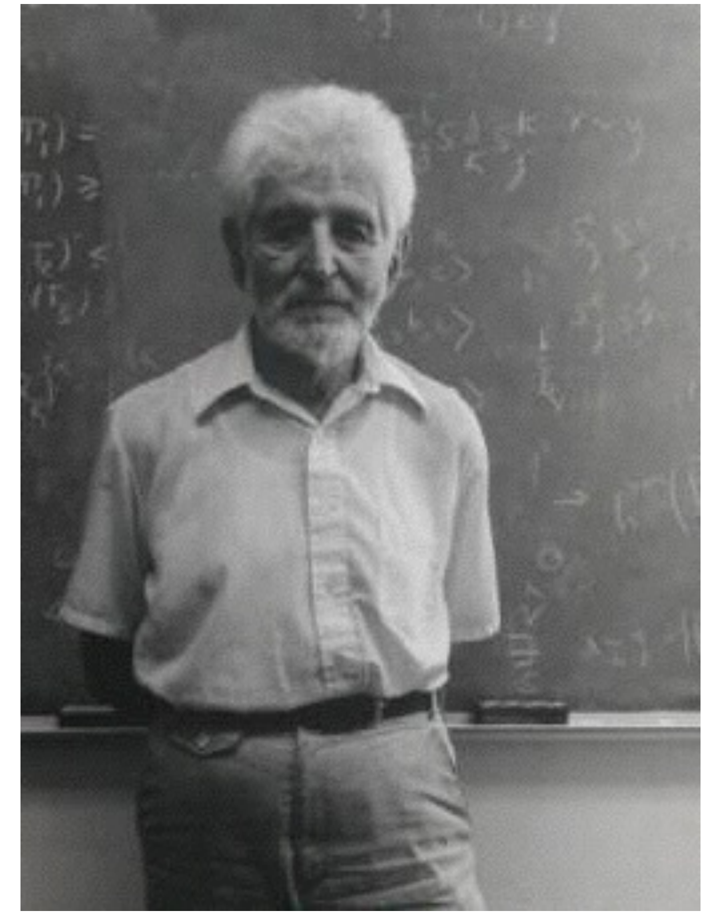
An interpolation theorem for first-order formulas with relational access restrictions

Balder ten Cate
LogicBlox & UC Santa Cruz

on the occasion of Johan van Benthem's retirement

Craig Interpolation



- **William Craig (1957)**: For all first-order formulas ϕ, ψ, if $\phi \vDash \psi$, then there is a first-order formula χ with $Voc(\chi) \subseteq Voc(\phi) \cap Voc(\psi)$ and $\phi \vDash \chi \vDash \psi$. Moreover the formula χ in question can effectively constructed from a proof of $\phi \vDash \psi$.

- Various extensions and variations have been proved (e.g., Lyndon interpolation, many-sorted interpolation, Otto interpolation).

- **Van Benthem (2008)**: "*Craig's Theorem is about the last significant property of first-order logic that has come to light. […] It seems fair to say that after Craig's theorem, no further significant properties of FOL have been discovered*."

Relational Access Restrictions

- A **database** is a (finite) relational structure over some schema $S = \{R_1, \ldots, R_n\}$
- **Relational access restrictions**: restrictions on the way we can access the relations $R_1, \ldots, R_n$.

First Example: View-Based Query Reformulation

- **Road network database**: Road(x,y)
- **Views**:
  - $V_2(x,y)$ = "∃ path of length 2 from x to y" = $\exists u\ \mathrm{Road}(x,u) \wedge \mathrm{Road}(u,y)$
  - $V_3(x,y)$ = "∃ path of length 3 from x to y" = $\exists u,v\ \mathrm{Road}(x,u) \wedge \mathrm{Road}(u,v) \wedge \mathrm{Road}(v,x)$
  - …
- **Observation**: V_4 can be expressed in terms of V_2.
- **Puzzle** (Afrati'07): can V_5 be expressed (in FO logic) in terms of V_3 and V_4?

Classic Results

- Querying using views has been around since the 1980s. E.g.,
- **Theorem** (Levy Mendelzon Sagiv Srivastava ’95): there is an effective procedure to decide whether a conjunctive query is rewritable as a conjunctive query over a set of views.
- **Open problem** (Nash, Segoufin, Vianu ‘10): is there an effective procedure to decide if a conjunctive query is answerable on the basis of a set of conjunctive views (a.k.a., is “determined” by the views)? if so, in what language can we express the rewriting?

Access Restrictions

- View-Based Query Reformulation:
  - *Can I reformulate Q as a query using only $V_1, \ldots, V_n$?*
  - *I.e., is Q equivalent to a query that only uses the symbols V1, …, Vn (relative to the theory consisting of the view definitions)?*
- Query Reformulation w.r.t. Access Methods (more refined):
  - *Can I find a plan to evaluate Q using only allowed access methods (possibly relative to some theory)?*
- First theory work by Chang and Li '01, followed by work of Nash, Ludaescher, Deutsch, …

Access Methods

- **Access method**: a pair (R,X) where R is an n-ary relation and X⊆{1, …, n} is a set of "input positions"
    - "*Relation R can be accessed if specific values are provided for the positions in X*."
- **Examples**:
    - (Telefoongids(name,city,address,phone#), {1,2})
    - (R,∅) means **free (unrestricted) access** to R.
    - (R,{1, …, n}) means only **membership tests** for specific tuples.
- There may be any number of access methods for a given relation.

Access Methods "Used" by a Query

- BindPatt(ϕ) is the set of access methods "used" by ϕ.

$$
\begin{array}{lcl}
\mathsf{BindPatt}(\top) \;=\; \mathsf{BindPatt}(x=y) & = & \emptyset \\
\mathsf{BindPatt}(R(t_1,\ldots,t_n)) & = & \{(R,\{1,\ldots,n\})\} \\
\mathsf{BindPatt}(\neg\varphi) & = & \mathsf{BindPatt}(\varphi) \\
\mathsf{BindPatt}(\varphi\wedge\psi) & = & \mathsf{BindPatt}(\varphi)\cup\mathsf{BindPatt}(\psi) \\
\mathsf{BindPatt}(\varphi\vee\psi) & = & \mathsf{BindPatt}(\varphi)\cup\mathsf{BindPatt}(\psi) \\
\mathsf{BindPatt}(\exists\vec{x}(R(t_1,\ldots,t_n)\wedge\varphi)) & = & \mathsf{BindPatt}(\varphi)\cup\{(R,\{i \mid t_i\notin\vec{x}\})\} \\
\mathsf{BindPatt}(\forall\vec{x}(R(t_1,\ldots,t_n)\to\varphi)) & = & \mathsf{BindPatt}(\varphi)\cup\{(R,\{i \mid t_i\notin\vec{x}\})\}
\end{array}
$$

- For example BindPatt(∀y(Rxy → Sxy)) = { (R,{1}), (S,{1,2}) }

- A "plan" for a query Q is a reformulation Q' of Q that only uses allowed access methods.

Motivation

- Query Reformulation w.r.t. Access Methods (more refined):
  - *Can I find a plan to evaluate Q using only allowed access methods (possibly relative to some theory)?*
- **Example***:* In the road network example, $V_5(x,y)$ admits a first-order plan only the access methods $(V_2,\varnothing)$ and $(V_3,\{1,2\})$.
- **Motivation***:*
  - Answering queries using data behind webforms.
  - Query optimization (*if a relation R(x,y) is stored in order sorted on x, access method (R,{2}) is much most costly than access method (R,{1}).*)
  - …

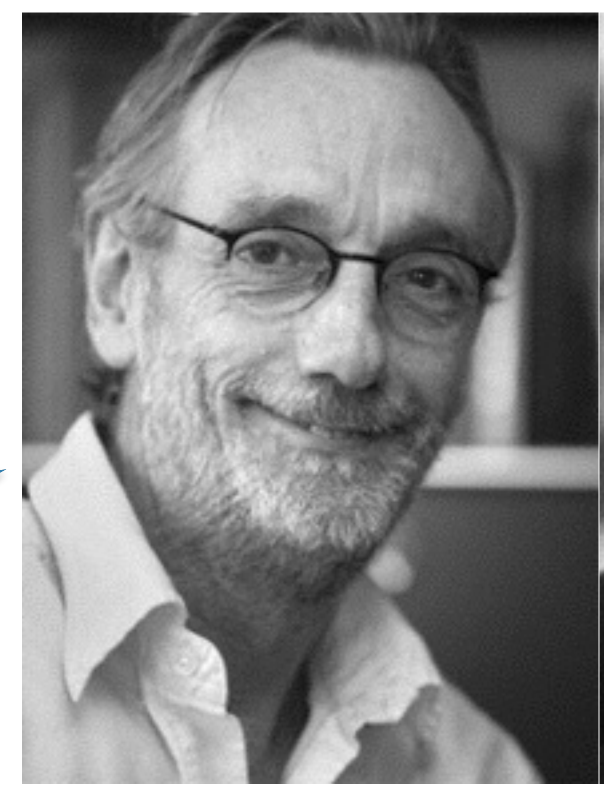
The Interpolation-Based Approach to View-Based Query Reformulation

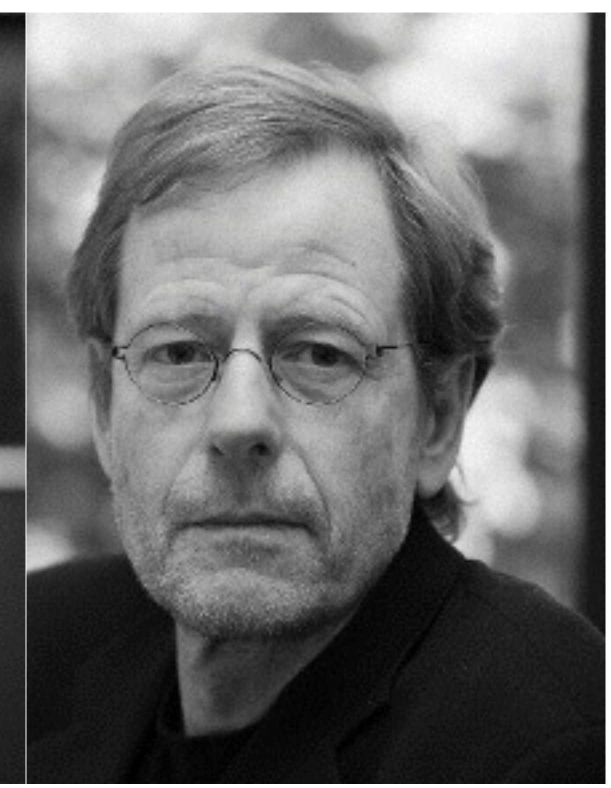
Key concepts

- **Determinacy**: V_4 is "determined by" (or "answerable from") V_2.
- **Query reformulations**: V_4 "can be reformulated as a query over V_2."

V_4 is
implicitly defined in
terms of V_2

$?xy.V_2(x,y)$
$\models$
$?xy.V_4(x,y)$





View-Based Query Reformulation

- Base relations $R_1 \ldots R_n$, view names $V_1 \ldots V_m$
- View definition theory: $T = \{ \forall \mathbf{x}(V_1(\mathbf{x}) \leftrightarrow \psi_1(\mathbf{x})), \ldots \}$, query Q
- The following are equivalent:
  1. Q is determined by $V_1 \ldots V_m$ (w.r.t. the theory T).
  2. a certain FO implication $\theta_{T,Q}$ is valid
  3. Q can be reformulated as a FO query over $V_1 \ldots V_m$. In fact, every Craig interpolant of $\theta_{T,Q}$ is such a reformulation.

What is going on?

- *From a proof of determinacy we are obtaining an actual reformulation.*
- This way of using interpolation to get explicit definitions from implicit ones goes right back to Craig's work.
- Same technique works for arbitrary theories T (not only view definitions).
- In principle this gives a method for finding query reformulations (but FO theorem proving is difficult).

- **Question**: can we do the same for the case with access methods?
- Answer: yes, using a suitable generalization of Craig interpolation.

Access Interpolation

- **Access interpolation theorem** (Benedikt, tC, Tsamoura, 2014): for all first-order formulas ϕ, ψ, if $\phi \vDash \psi$, then there is a first-order formula χ with BindPatt(χ) $\subseteq$ BindPatt(ϕ) $\cap$ BindPatt(ψ) and $\phi \vDash \chi \vDash \psi$. Moreover the formula χ in question can effectively constructed from a proof of $\phi \vDash \psi$.

- Can be further refined by distinguishing positive/negative uses of binding patterns.

- Generalizes many existing interpolation theorems (Lyndon, many-sorted interpolation, Otto interpolation)

- Gives rise to a way of testing "access-determinacy" and the existence of reformulations w.r.t. given access methods, as well as a method for finding such reformulations.

Examples in Mathematical Logic

- In set theory, a Δ_0-formula is a formula that only uses access method ($\in$, {2})
- In bounded arithmetic, we study formulas that only use access method ($\leq$, {2}).
- The access interpolation theorem generalizes an interpolation theorem for "$\leq$-persistent" formulas by Feferman (1967).

Summary

Querying under Access Restrictions

1. View-based query reformulation (restricting to a subset of the signature)

   This is the setting of the (projective) Beth theorem. We look for a proof of implicit definability ("determinacy") and, from it, compute an explicit definition ("query reformulation") using Craig interpolation.

2. Query reformulations given access methods (more refined)

   Same general technique applies, using a suitable adaptation of Craig interpolation: access interpolation.

Three Important Subtleties

1. Databases are finite structures. But Craig interpolation for first-order logic fails in the finite.

2. For practical applications, we need effective algorithms. But first-order logic is undecidable (we cannot effectively decide if the implication $\theta_{T,Q}$ is valid).

3. For practical applications, we don't want just any query reformulation, we want one of low cost.

Solutions

- The solution for 1 and 2 is to **move to a fragment** of first-order logic that is **decidable** and that has the **finite model property**, while still being sufficiently expressive.
- Natural candidate: the guarded fragment.

Guarded Fragment

(Andreka, van Benthem, Nemeti 1998)

- All quantification must be guarded.

$$\phi ::= R(x_1 \ldots x_n) \mid x=y \mid \neg\phi \mid \phi\wedge\phi \mid \exists \mathbf{y}.G(\mathbf{x,y,z})\wedge\phi(\mathbf{x,y,z})$$

- GF has become an extremely successful and well studied fragment of first-order logic.
- Inherits all the good properties of modal logic (robust decidability, finite model property, …)
- Except Craig interpolation.

Guarded Negation Fragment

- Guarded-Negation fragment (GNFO): a slight further extension of the guarded fragment that does have Craig interpolation.

- Instead of guarding quantifiers we guard the negation.

  $\phi ::= R(x_1 \ldots x_n) \mid x=y \mid G(\mathbf{x}) \wedge \neg\phi(\mathbf{x}) \mid \phi \wedge \phi \mid \exists \mathbf{y}.\phi$

  - Note: sentential and unary negation can be trivially guarded by the identity guard x=x.

- GNFO retains all the good properties of GF (Barany, tC, Segoufin 2011),

- It also has (effective) Craig interpolation (Barany, Benedikt, tC 2013; Benedikt, tC, Vanden Boom 2014)

Cost-sensitive Query Reformulation

- Every real-world database management system has a cost-estimate function for query plans (what is the expected execution time).
- We are looking for a proof of $\theta_{T,Q}$ such that the interpolant obtained from it constitutes a plan that has a low cost.
- Idea: explore the space of possible proofs *guided by (monotone) plan cost function.*
- Under suitable restrictions, it is possible to obtain cost-optimal plans this way.
- Ongoing research, in collaboration between Oxford University (Michael Benedikt) and LogicBlox.
- There will be openings for postdocs at Oxford on this.

Thank you

Solution

- $V_5(x,y) = \exists u\ (\ V_4(x,u) \wedge \forall v\ (\ V_3(v,u) \rightarrow V_4(v,y)\ )\ )$