Events or indices*

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Abstract

The paper discusses the question of whether events should be used as part of formal semantic systems aiming at describing temporal structure in natural language or not, the alternative being to use numbers. In particular, it will be argued that an approach which aims at tying up semantic information with number systems is much simpler than a system based on the Russell-Wiener-construction in which time is derived from events. In fact, the notion of event is essentially a macro-notion useful at the level of discourse analysis, not at the micro-level in which predication is to be decomposed into its constituent parts.

Contents

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1 Introduction

In this paper, I would like to shed some light on the question of whether events should be used as part of formal semantic systems for describing temporal structure in natural language or not, the alternative being to use numbers. In my own work on aspectuality, I started as far back as 1968 to use events as part of the analysis of temporal structure, following Davidson 1980, also because he built on Reichenbach's marvellous chapter on the analysis of conversational language, in particular on the problem of individuals. Gradually it occurred to me that events are not useful for the analysis of aspectual composition if we take them the way Davidson does. At least I found no way to apply his insights to my work on aspectuality.

Davidson's analysis of events is essentially based on a macro-perspective which turned out to be useful in the study of discourse: events have a structural role to play in the construction of discourse. For the analysis of the compositional mechanism bringing about aspectuality, however, one may argue, as I have been doing, for the need of a micro-perspective in which events are taken as being construed. As a result of my growing scepsis about events as explanatory tools, they have been replaced by numbers in my system. Maybe some do not find the difference important, but I think it is, if only because a lot of suspect ontology has penetrated into linguistic considerations. In my view, it is numbers and not events that play a role in the contraction and expansion mechanism employed by speakers of natural languages.[1] The use of natural numbers is necessary to provide us with a dimension of discreteness, the use of the real numbers is necessary to model our experience with three dimensional space. A telling metaphor is provided by traveling in the metro: we count our progress in terms of the stations, in $\mathbb{N}$, while we experience our real progress, of course, in terms of the distance we have covered, which has the structure of the reals $\mathbb{R}$.

In Verkuyl 1978, this notion of $\mathbb{N}$-to-$\mathbb{R}$-shift was developed with the help of an example in which the white queen is moved on the chess board from d2 to d7. Two perspectives play a role here: the physical move along the densely organized Path on the board itself, in casu the $\mathbb{R}$-interval [d2,d7], and the move of the game where the fields are numbered by the indices d2, d3, ..., d7, discretely organized in $\mathbb{N}$. In our use of natural language we constantly shift back and forth between the systems. I think that the introduction of events in our theoretical language prevents us from focussing on this shifting mechanism. Accordingly, I would like to reduce the naive physics popping up in so many event semantic contributions as much as possible in favour of a much more abstract organization of the domain of discourse.

If this is the first sketch of the right picture, then the question arises where the notion of event comes in. My answer is: quite late. The term *event* is useful, but only as a descriptive term, convenient and abbreviatory. For the analysis of our dealing with the interaction between atemporal and temporal structure

[1] I distinguish here between Naturals and Reals, without bothering about whether the Integers and the Rationals could do the job as well. I side here with van Benthem 1983.

we have to aim at theoretically useful notions. We can handle numbers very well which means that the use of the notion of event can be postponed up to the "performance level", the level at which events are actualized in real time. As a slogan for languages with tense: no events without tense. In the present paper, I would like to promote this view by a critical investigation of the model theory of Kamp and Reyle 1993:664–79.

Kamp & Reyle (from now on K&R when I refer to the authors) discuss three options for the model structure which is part of their model theory. The first option is to derive the notion of event from a more fundamental notion of time. They reject this option in favour of the second option of defining time in terms of events and in so doing they make use of the so-called Russell-Wiener construction, which also was discussed in Kamp 1979 and Kamp 1980. Later on, however, they turn to a third option, on which no reduction is necessary because both events and time are "out there". But in this option they maintain the spirit of Russell and Wiener concerning the relation between event structure and time structure.

A possible difference between the Russell-Wiener approach which will be sketched shortly and my approach might be, according to Johan van Benthem (pers. comm.), that Russell and Wiener considered mathematics a rather late invention in the development of mankind, which means that numbers as objects of investigation came quite late in its history. My plea for giving numbers a very important place in the semantic analysis of natural language is motivated by the conviction that numbers are deeply involved in the computational machinery that is necessary for having a grammar. For example, the recursive mechanism underlying grammar providing it with the force to produce an infinite number of sentences, may be taken as an indication that in the organization of our cognitive capacities the role of the (discrete) number systems is important indeed. My contention is that also in interpreting language an appeal is made to the number systems in order to be able to make the information conveyed by sentences manageable. Especially the way in which we switch, say, from speaking about France as a country in which you can stay to France as a member of the European community having one vote, makes it plausible to assume an ability to jump from the reals into the naturals and back. Van Benthem's suggestion is that K&R did not want to break with the Russellian tradition in which natural language was considered poor rather than equipped with sophisticated logical machinery. Against this possible background I would like to reduce the tripartite perspective inherent to the Russell-Wiener construction by showing that the aspectual Path structure based on the idea of an $\mathbb{N}$-to-$\mathbb{R}$-shift meets nearly all the postulates that are said to underlie event structure. At the point where a difference is visible, it seems more natural to rely on numbers rather than on events. Thus it seems that an approach which aims at tying up semantic information with number systems is much simpler than a system based on the Russell-Wiener-construction.

2 A tripartite model structure: Russell-Wiener

2.1 Three model structures

After having rejected, in Chapter 5.1 of Kamp and Reyle 1993, the option of defining events in terms of times, K&R discuss the remaining two options in Chapter 5.6. For the second option they describe the so-called Russell-Wiener construction in which the notion of time is derived from the primitive notion of event. The construction consists of two steps and involves three sorts of model structure. Events give rise to instants and instants are used to get at intervals in real time. The construction distinguishes between an event structure $\mathcal{E}$, an instant structure $\mathcal{I}(\mathcal{E})$ and an interval structure $Int(\mathcal{I}(\mathcal{E}))$, as shown in Figure 1.

$$\begin{tikzcd} \mathcal{I}(\mathcal{E}) \arrow[dr, "Int"] & \\ \mathcal{E} \arrow[u, "\mathcal{I}"] \arrow[r, "\mathsf{p}"] & Int(\mathcal{I}(\mathcal{E})) \end{tikzcd}$$

(a)

$$\begin{tikzcd} \mathsf{I}(\mathcal{E}) \arrow[d, "\mathsf{I}(\mathcal{E}) \subseteq \wp(\mathsf{E})"'] & \\ \mathsf{E} & \mathsf{Int} \arrow[ul, "\mathsf{Int} \subseteq \wp(\mathsf{I}(\mathcal{E}))"'] \end{tikzcd}$$

(b)

Figure 1: Russell-Wiener according to K&R.

Figure 1a shows the model structures mentioned in Kamp and Reyle 1993:667–9, Figure 1b gives the sets of these structures: the set of events E, the set $\mathsf{I}(\mathcal{E})$ of instants and the set Int of convex subsets derived from an instant structure $\mathcal{T} = \langle \mathsf{T}, < \rangle$. The up-arrow in Figure 1a shows the operation on events as provided by the K&R-definition 5.6.1 (I maintain their numbering), which produces an instant structure at the power set level. In terms of sets and illustrated in Figure 1b, this means that $\mathsf{I}(\mathcal{E}) \subseteq \wp(\mathsf{E})$, an instant being defined as a subset of E. The instant structure $\mathcal{I}(\mathcal{E})$ is taken by K&R as a substructure of $\mathcal{T}$. In this sense, the set $\mathsf{I}(\mathcal{E})$ is supposed to mediate between events E and times from T. In fact, $\mathsf{Int} \subseteq \wp(\mathsf{T})$, with T the set of instants (points) constituting the time axis and $<$ a strict partial order, in fact a linear order. This means that we also have $\mathsf{Int} \subseteq \wp(\mathsf{I}(\mathcal{E}))$. At least this should be concluded from a remark K&R make on page 671 where they treat the third option mentioned above. They say there that they want to "retain as much of the spirit of the second option as possible [by assuming] that the 'instant' structure $\mathcal{I}(\mathcal{E})$ is a substructure of $\mathcal{T}$." So, in general the arrows in Figure 1a can be understood as follows: the structure at the arrow-head is in some way dependent on the structure at the origin. In this sense both $\mathcal{E}$ and $\mathcal{T}$ can be said to be generative. They meet, so to say, at the beginning of the diagonal arrow. In particular, the instant structure $\mathcal{I}(\mathcal{E})$ is really the point at which the transition from event to time is considered to take place. The bottom arrow represents a homomorphism p which assigns to each event e in E a corresponding value $\mathsf{p}(\mathsf{e}) = \{\mathsf{i} \in \mathsf{I}(\mathcal{E}) : \mathsf{e} \text{ occurs at } \mathsf{i}\}$.

Figure 1b has been added because it is necessary to see what K&R do if they describe the Wiener-Russell construction. The arrows indicate the direction of the set-theoretical relation between the sets involved. If $\mathcal{I}(\mathcal{E})$ is indeed a substructure of $\mathcal{T}$, it should follow that the set Int is a subset of the powerset of $\mathsf{I}(\mathcal{E})$, because Int is also to be taken as a subset of the power set of T: $\mathsf{Int} \subseteq \wp(\mathsf{T})$. In this way, we can understand the homomorphic mapping p as a mapping of $\mathcal{E}$ into $\wp(\mathcal{T})$. Which is, in fact, the option K&R end with.

2.2 Event Structure

Having established now that the crucial point where the notion of event transmutes into the notion of time unit is $\mathcal{I}(\mathcal{E})$, we need to inspect the definitions involved in order to see what exactly happens. It will be argued in due course that the triangle can be reduced to a function $Int : \mathsf{I} \longrightarrow \mathsf{Int}$, and therefore it is necessary to execute this inspection in some detail. Here are the postulates governing $\mathcal{E}$, with $<$ for the relation of precedence and $\circ$ for the relation of overlap.[2]

(1)	P1.	$\mathsf{e}_1 < \mathsf{e}_2 \Rightarrow \neg \mathsf{e}_2 < \mathsf{e}_1$	Asymmetry
	P2.	$\mathsf{e}_1 < \mathsf{e}_2 \wedge \mathsf{e}_2 < \mathsf{e}_3 \Rightarrow \mathsf{e}_1 < \mathsf{e}_3$	Transitivity
	P3.	$\mathsf{e} \circ \mathsf{e}$	Reflexivity
	P4.	$\mathsf{e}_1 \circ \mathsf{e}_2 \Rightarrow \mathsf{e}_2 \circ \mathsf{e}_1$	Symmetry
	P5.	$\mathsf{e}_1 < \mathsf{e}_2 \Rightarrow \neg \mathsf{e}_2 \circ \mathsf{e}_1$	
	P6.	$\mathsf{e}_1 < \mathsf{e}_2 \wedge \mathsf{e}_2 \circ \mathsf{e}_3 \wedge \mathsf{e}_3 < \mathsf{e}_4 \Rightarrow \mathsf{e}_1 < \mathsf{e}_4$	
	P7.	$\mathsf{e}_1 < \mathsf{e}_2 \vee \mathsf{e}_1 \circ \mathsf{e}_2 \vee \mathsf{e}_2 < \mathsf{e}_1$	Linearity

K&R do not bother to explain the notion of $\circ$ at the intuitive level: they rely on (1). As a matter of fact, Kamp 1979 defines overlap as in (2), given a temporal instant structure $\mathcal{T} = \langle \mathsf{T}, < \rangle$.

(2) $\mathsf{e}_1 \circ \mathsf{e}_2 =_{df} \exists t(t \in \mathsf{e}_1 \wedge t \in \mathsf{e}_2)$

Here the notion of event is made time-dependent by the very definition of overlap in terms of the time-axis. It follows that the primitive notion of event is not primitive after all. However, it is not sure whether or not K&R maintain (2), so I will not use it here as an argument against their use of the Russell-Wiener construction. In general, though, (2) expresses the idea of an event having length: it would be quite misleading to think of e_1 and e_2 as singletons.

Whatever we may observe about the set of postulates in (1), at least we should say that the notion of overlap is not clearly defined by it. That is, the intuition about what an overlap is, cannot be "read" from the postulates P3-P7. The sense of length which is clearly to be evoked by the notion of an e is not expressed in the postulates themselves. This can be seen by simply replacing $\circ$ by $=$. The postulates P3 - P7 would hold even though the relation $=$ would lead to transitivity, $\circ$ being not transitive, because of $\mathsf{e}_1 \circ \mathsf{e}_2 \wedge \mathsf{e}_2 \circ \mathsf{e}_3 \not\Rightarrow \mathsf{e}_1 \circ \mathsf{e}_3$.

One may push this point a little further because in the present context there is a natural tie between $\circ$ and $=$. The former is purported to overcome the limits of $=$ because for events one does not want to have transitivity and that is exactly why $\leq$ is considered inadequate. This suggests that the question

[2]They are discussed in Kamp 1979 and in Kamp 1980.

of whether or not one needs events boils down to the question of whether the extension of $\leq$ into $<$ plus $\circ$ is necessary.

2.3 Instant Structure

Given the postulates P1 - P7, K&R "generate" the instant structure $\mathcal{I}(\mathcal{E})$ by the following definition, which I shall label as it is labeled by K&R:

Definition 5.6.1. Let $\mathcal{E}$ be an event structure as constrained by P1 - P7. Then $\mathcal{I}(\mathcal{E}) = \langle \mathsf{I}, <_{\mathsf{i}} \rangle$, with I the set of instants i as defined in (i) and $<_{\mathsf{i}}$ the precedence relation as defined in (iii):

(i) i is an instant of $\mathcal{E}$ if
  a. $\mathsf{i} \subseteq \mathsf{E}$;
  b. $\mathsf{e}_1, \mathsf{e}_2 \in \mathsf{i} \Rightarrow \mathsf{e}_1 \circ \mathsf{e}_2$
  c. if $\mathsf{H} \subseteq \mathsf{E}, \mathsf{i} \subseteq \mathsf{H},$ and for all $\mathsf{e}_1, \mathsf{e}_2 \in \mathsf{H}, \mathsf{e}_1 \circ \mathsf{e}_2,$ then $\mathsf{H} \subseteq \mathsf{i}$

(ii) An event e *occurs at* an instant i iff $\mathsf{e} \in \mathsf{i}$

(iii) For all instants $\mathsf{i}_1, \mathsf{i}_2 : \mathsf{i}_1 <_{\mathsf{i}} \mathsf{i}_2$ iff there are $\mathsf{e}_1 \in \mathsf{i}_1, \mathsf{e}_2 \in \mathsf{i}_2$ such that $\mathsf{e}_1 < \mathsf{e}_2$

It is important to see that by the a-clause of (i) i is defined as a set of events e. K&R are specific about that: membership of an event e in i means that e is going on at i. The b-clause of (i) says that for any i there must be a pairwise overlap between any two members of i. The c-clause introduces the maximality of i. As a result an instant is a maximal set of pairwise overlapping events. An element of i "occurs" at i. Here we see the transmutation into temporality, because at the next stage of the construction the instant structure is taken as a substructure of $\mathcal{T}$, the structure involving the time axis. Clause (iii) is interesting because the notion of precedence is now going to deviate from the notion of precedence holding between events. This is illustrated in Figure 2, where i_1 i-precedes i_2, because e_1 e-precedes e_3. Here the essence of the Russell-Wiener construction

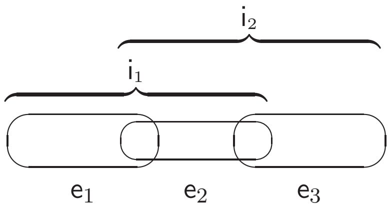

Figure 2: $\mathsf{i}_1 <_{\mathsf{i}} \mathsf{i}_2$

becomes visible: i-precedence makes it possible to consider the set $\{\mathsf{e}_1, \mathsf{e}_2\}$ as a set preceding the set $\{\mathsf{e}_2, \mathsf{e}_3\}$, in some natural sense of precedence, namely in the sense of ordering even though there is overlap between the sets.

2.4 Interval Structure

On top of $\mathcal{I}(\mathcal{E})$ a new structure is constructed which is called $\mathit{Int}(\mathcal{I}(\mathcal{E})) = \langle \mathsf{Int}, <_{\mathsf{p}}, \circ_{\mathsf{p}} \rangle$. As $\mathit{Int}(\mathcal{I}(\mathcal{E}))$ is a substructure of $\mathcal{T}$, this means that Int is a set of convex subsets of T, i.e. a set of intervals. The relations $<_{\mathsf{p}}$ and $\circ_{\mathsf{p}}$ are defined on the basis of the K&R-definition 5.6.2.

Definition 5.6.2. Let X, Y be intervals of the instant structure $\mathcal{P}(\mathcal{E})$. Then:

(i) $\mathsf{X} <_{\mathsf{p}} \mathsf{Y}$ iff for all $\mathsf{i}_1 \in \mathsf{X}$ and $\mathsf{i}_2 \in \mathsf{Y}, \mathsf{i}_1 < \mathsf{i}_2$
(ii) $\mathsf{X} \circ_{\mathsf{p}} \mathsf{Y}$ iff $\mathsf{X} \cap \mathsf{Y} \neq \emptyset$
(iii) $\mathsf{X} \subseteq_{\mathsf{p}} \mathsf{Y}$ iff for every instant $\mathsf{i} \in \mathsf{X}, \mathsf{i} \in \mathsf{Y}$

This definition can be illustrated by considering the following sets:

(3) a. $\mathsf{X} = \{\{_{\mathsf{i}_1}\mathsf{e}_1, \mathsf{e}_2\}, \{_{\mathsf{i}_2}\mathsf{e}_2, \mathsf{e}_3\}\}$ $\quad$ $\mathsf{Y} = \{\{_{\mathsf{i}_3}\mathsf{e}_4, \mathsf{e}_5\}, \{_{\mathsf{i}_4}\mathsf{e}_6, \mathsf{e}_7\}\}$
b. $\mathsf{X} = \{\{_{\mathsf{i}_1}\mathsf{e}_1, \mathsf{e}_2\}, \{_{\mathsf{i}_2}\mathsf{e}_2, \mathsf{e}_3\}\}$ $\quad$ $\mathsf{Y} = \{\{_{\mathsf{i}_3}\mathsf{e}_3, \mathsf{e}_4\}, \{_{\mathsf{i}_4}\mathsf{e}_5, \mathsf{e}_6\}\}$

In (3a), X clearly precedes Y: the definition precludes overlap. Note in passing that in (3b), e_3 is part of both X and Y, but it should not count. What counts is overlap of instants. In view of the mapping p, I tend to think that (3b) should be excluded, but Definition 5.6.1 does not exclude it: overlap of two events e does not require that they should form an instant.

There is overlap in (4a) and overlap and inclusion in (4b):

(4) a. $\mathsf{X} = \{\{_{\mathsf{i}_1}\mathsf{e}_1, \mathsf{e}_2\}, \{_{\mathsf{i}_2}\mathsf{e}_2, \mathsf{e}_3\}\}$ $\quad$ $\mathsf{Y} = \{\{_{\mathsf{i}_2}\mathsf{e}_2, \mathsf{e}_3\}, \{_{\mathsf{i}_3}\mathsf{e}_3, \mathsf{e}_4\}, \{_{\mathsf{i}_4}\mathsf{e}_4, \mathsf{e}_5\}\}$
b. $\mathsf{X} = \{\{_{\mathsf{i}_1}\mathsf{e}_1, \mathsf{e}_2\}, \{_{\mathsf{i}_2}\mathsf{e}_2, \mathsf{e}_3\}\}$ $\quad$ $\mathsf{Y} = \{\{_{\mathsf{i}_1}\mathsf{e}_1, \mathsf{e}_2\}, \{_{\mathsf{i}_2}\mathsf{e}_2, \mathsf{e}_3\}, \{_{\mathsf{i}_3}\mathsf{e}_3, \mathsf{e}_4\}\}$

The examples in (3) and (4) illustrate the nature of the mapping between the structures $\mathcal{E}$ and $Int(\mathcal{I}(\mathcal{E}))$. This is done by the homomorphism p, which is structure-preserving with respect to $<$ and $\circ$. What it does is to provide for e_1 in (3a) and (4) the set $\{\{\mathsf{e}_1, \mathsf{e}_2\}\}$ and for e_2 the set $\{\{\mathsf{e}_1, \mathsf{e}_2\}, \{\mathsf{e}_2, \mathsf{e}_3\}\}$, if we follow K&R. In this way an event is associated with the instants at which it occurs.

This finishes the description of the second option discussed in Kamp & Reyle. Even though they end up by adopting a third option, it was necessary to give a detailed description of the second one, because some of the structures in their final choice are defined as above: "in order to retain as much of the spirit of the second option as possible we assume that the 'instant structure' $\mathcal{I}(\mathcal{E})$ is a substructure of $\mathcal{T}$". Moreover, my alternative to their final choice makes use of some of the machinery provided by the two definitions.

2.5 Events and Times

The third option discussed by K&R is to abandon the idea that times should be definable from events, which means that both events and times are taken as primitive categories, that time has the structural properties of the reals $\mathbb{R}$, and that the event structure and time structure are connected by some structure preserving function, which is called LOC. I will not go into all the details of this option because it suffices to follow the components of the model structure specified by K&R. They have:

- an event structure $\mathcal{E} = \langle \mathsf{E}, <, \circ \rangle$ as defined above.
- a linearly ordered and compact time structure $\mathcal{T} = \langle \mathsf{T}, < \rangle$.

- a function $\text{LOC}: \mathcal{E} \longrightarrow \mathcal{T}'$ defined by:
  (a) if $\mathsf{e} \circ \mathsf{e}'$, then $\text{LOC}(\mathsf{e}) \cap \text{LOC}(\mathsf{e}') \neq \emptyset$
  (b) for every $\mathsf{i} \in \mathcal{I}(\mathcal{E}) : \bigcap\{\text{LOC}(\mathsf{e}) : \mathsf{e} \in \mathsf{i}\} \neq \emptyset$
  where $\mathcal{T}'$ is a homomorphic contraction of $\mathcal{T}$.

K&R underscore the need to "require of each model that its instant structure $\mathcal{I}(\mathcal{E})$ instantiates the conception of time modern physics requires (e.g. the conception that time is isomorphic to the real number structure $\mathbb{R}$)" (p.670). The most appropriate way of illustrating the third option may be Figure 3, in which time and events are treated separately, the only direct connection being

$$\begin{tikzcd} \mathcal{I}(\mathcal{E}) & & \mathcal{T} \arrow[dd] \\ & Int & \\ \mathcal{E} \arrow[uu, "\mathcal{I}"] \arrow[rr, "\text{LOC}"] & & Int(\mathcal{T}) \end{tikzcd}$$

(a)

$$\begin{tikzcd} \mathsf{I}(\mathcal{E}) \arrow[dd, "\mathsf{I}(\mathcal{E}) \subseteq \wp(\mathsf{E})"] & {[\subseteq]} & \mathsf{T} \\ & & \\ \mathsf{E} & & \mathsf{Int} \arrow[uu, "\mathsf{Int} \subseteq \wp(\mathsf{T})"] \end{tikzcd}$$

(b)

Figure 3: The third option

the LOC-function. Whatever the merits of this third option, it is clear that it crucially uses the main ingredients of the Russell-Wiener construction, the difference being that rather than construing time from events there is now an isomorphic mapping between event structure and time structure. Because the third option retains the merits of the second option, as well as its spirit, I will couch a comparison between the event approach and the number approach in terms of the Russell-Wiener machinery described above.

3 Path Structure and the Naturals

The basic notion underlying the so-called localistic approach is that in the domain of interpretation of sentences expressing a change, there is a Path along which the change expressed by the predicate takes place. The localistic tradition in linguistics is very old, but in the mid-sixties it was revived by the work collected in Gruber 1976. All sorts of change can be modelled with the help of an abstract notion of Path. For example, in *John got angry* John "moved" along a (metaphorical) Path in the form of a sort of scale so that he ended up in the area of anger; in *Judith ate four sandwiches* the way in which Judith is involved in the predication can be seen as a way of "going through it" until the four sandwiches are eaten, etc. The localistic notion of Path plays an important role in my 1971-thesis, which appeared as Verkuyl 1972. Important elements of it are the adding-to increase of information along the Path (some call it cumulativity nowadays) and the dependency of the movement expressed by the verb on the information provided by the argument (some call it measuring out nowadays). It should be observed that around 1970 formal semantics was quite unknown, so whatever was proposed, was clothed in generative terms, which from the model-theoretic point of view are clearly insufficient.

In the early eighties, I embarked upon a model-theoretic interpretation of the 1971-system, aiming at defining the Path-notion in set-theoretical terms. After all, in sentences expressing change there is some dynamics involved, so the question became: what happens if you are "going through" a set? In terms of set theory, this is an odd question but the idea itself is not so odd. When we interpret the VP in sentences like (5),

(5) a. Judith ate four sandwiches
    b. The three girls wrote some letters

it contains an atemporal unit ($[\![$four sandwiches$]\!]$, $[\![$some letters$]\!]$) which in some way is involved in the development of temporal structure introduced by the verb. So, what happens set-theoretically if you relate an atemporal set to a linear structure? This question is not so odd any longer, the more so while you can send a line through a set relating its elements to it by partitioning the set, as illustrated in Figure 4, where (a) shows the situation at which the verb and its argument are going to be structurally related into the VP; and where (b) illustrates the effect of amalgamating the verbal and nominal information into a Path structure. Sentence (5a) expresses a predication. The natural

Figure 4: Relating sets to linear order

computational thing to do concerning the satisfaction of the truth conditions is to establish how the elements of the set S of four sandwiches were involved in the predication. Figure 4 illustrates an equivalence relation C as determining an arbitrarily chosen partition P with three blocks (or cells): $P = [S]_C$.[3] The equivalence relation can be described as 'counting as being involved in the predication'. Indices as occurring in Figure 4b, which are contributed by the verb, come in quite naturally as providing a well-ordering: the set $\{s_1\}$ formed at counting-point 1 is followed by the set $\{s_2, s_3\}$ formed at counting-point 2, etc. In other words, the interpretation of the VP *eat four sandwiches* invokes the principle of mathematical induction. The interpretation contains information about the merger of the order provided by the verb and the partitioned internal argument. Formally, the VP may be taken as denoting a function $\lambda x.\ell_x$ applying to values x in the external argument domain and yielding for each x its Path ℓ_x as defined in Definition 3.1:

Definition 3.1

[3] In this sense Figure 4 is a simplification: sentences like (5a) present a set of configurational possibilities of which the one in Figure 4 is just one of many. Cf. Verkuyl 1993 for a detailed explanation of the mechanism involved.

$$\ell_x = \{\langle \mathsf{i}, y\rangle : [\![\text{AT}]\!]_{\mathrm{M},\mathsf{i}}(y)(x) = 1\}$$

That is, for each element x of the external argument domain one can determine a set of pairs such that for each $\mathsf{i} \in \mathsf{I}$, x satisfies at i the y-part of the predication. Technically, y is a block of a partition. From the localistic point of view, y is the position AT which the external argument is on its way of satisfying the predication with respect to the internal argument: to make a predicate true its arguments go through a set of "positions". In the case of Figure 4b, the function $\ell_j = \{\langle 1, s_1\rangle, \langle 2, s_{2,3}\rangle, \langle 3, s_4\rangle\}$. In this way, the Path of Judith through eating the sandwiches follows a particular partition. However, Judith may have eaten her sandwiches one-by-one or in groups or all at once. The definition in (3.1) does not provide a particular partition, but for any external argument member there is a Path, structured by the definition. Modifiers may give information about the way a Path is structured.

The subscripts on the function ℓ are explained by the need to properly deal with plural external arguments. The copying effect of multiplication is automatically achieved by the lambda-function itself: it provides a set of functions ℓ_x in the case of a plural external argument. In (5a), $\lambda x.\ell_x(j) = \ell_j$, but in (5b) we obtain:

$$\begin{aligned}
&\lambda x.\ell_x(girl_1) = \ell_{girl_1}\\
&\lambda x.\ell_x(girl_2) = \ell_{girl_2}\\
&\lambda x.\ell_x(girl_3) = \ell_{girl_3}.
\end{aligned}$$

I ignore here the distinction made between the so-called injective and constant modes which taken as constraints on the application of $[\![\mathrm{VP}]\!]$. The injective mode requires different Paths for each of the members of the internal argument, on the constant mode all its members are mapped onto the same ℓ.

The localism involved in the account of change can be tied up very naturally with the number systems: a so-called [+ADDTO]-verb like *eat* is interpreted as introducing a well-ordered set I of indices i, taken as natural numbers. By Definition 3.2 indices in I are the endpoints of intervals in $\mathbb{R}$, the set of real numbers.

Definition 3.2

$$I_V := \{(0, \mathsf{k}) \subseteq \mathbb{R} | \mathsf{k} \in \mathbb{N}\}$$

The successor function $s : \mathsf{I} \longrightarrow \mathsf{I}$ is defined by $\forall \mathsf{k} \in \mathsf{I} : s(\mathsf{k}) = \mathsf{k} + 1$.[4] The connection between I and I_V is made by a function $succ : \mathsf{I}_V \longrightarrow I_V$ defined as: $\forall \mathsf{k} \in \mathbb{N} : succ((0, \mathsf{k})) = (0, s(\mathsf{k}))$. Intuitively, this definition creates a sense of progress in which the point of origin 0 is fixed. It provides filter structure in the set of intervals making up a Path. This solves the traditional problem with Von Wright's operator T (Cf. Kamp 1980 for a discussion about it). Finally, it expresses the fact that underlying the notion of Path structure the system of natural numbers is constructed set-theoretically, as I will point out shortly.

[4]The reason for distinguishing I from $\mathbb{N}$ is that I might be taken as a set isomorphic to $\mathbb{N}$ except for the property of equidistance, as argued in Verkuyl 1987.

This concludes the presentation of the formal machinery defining Path structure. In the next section, I will show that the model structure $\mathcal{I}(\mathcal{E})$ has the same properties as the model structure involved in mapping indices onto atemporal sets. In fact, it will be shown that apart from some intriguing deviation from the postulate P5, the model structure $\mathcal{I} = \langle \mathsf{I}, < \rangle$ underlying the Definition 3.1 has the same structure as K&R's $\mathcal{I}(\mathcal{E})$, without being dependent on the existence of $\mathcal{E}$.

4 Path Structure as Instant Structure

Figure 4b is convenient to illustrate the formal machinery to be presented as an alternative to the Wiener-Russell construction. A Path is defined as a set of pairs, the first members of which form a well-ordered series of indices $\{0, 1, 2, \ldots\}$. Information about the internal argument is associated with each index by the function ℓ. This function is defined cumulatively (For the details see Verkuyl 1987; 1993) This means that an index i_k should be seen as including i_{k-1}, in the same sense in which having eaten three sandwiches entails that you have eaten two sandwiches. So, precedence can be defined in terms of the number system in which the well-orderedness of the naturals are a way to capture this sort of precedence.

This can be obtained by construing natural numbers as sets, while making use of their status as individuals, in the way demonstrated in for example Partee et al. 1990:75f. which sketches a method to treat the numbers $0, 1, 2, 3, \ldots$ as sets by defining them as in (6).

(6) $0 =_{\text{df}} \emptyset$
$1 =_{\text{df}} \{\emptyset\} = \{0\}$
$2 =_{\text{df}} \{\emptyset, \{\emptyset\}\} = \{0, 1\}$
$3 =_{\text{df}} \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} = \{0, 1, 2\}$
$4 = \ldots$

This Fregean definition provides us with an interesting double perspective. The fourth line of (6) says that $3 = \{0,1,2\}$. In other words, the number 3 is taken as a set with 0, 1 and 2 as its elements. In order to make it possible to compare the K&R-approach with the Path approach, I will write the numbers now as subscripts on i and e. We obtain then $\mathsf{i}_3 = \{\mathsf{e}_0, \mathsf{e}_1, \mathsf{e}_2\}$, which should be understood as simply saying that $3 = \{0, 1, 2\}$. The only goal of the rewriting is to underscore the obvious parallel with an instant i as a set of e's as defined in Definition 5.6.1 and illustrated in Figure 2. There the relation between instants and events was clothed in terms of the $\in$-relation. Here, in (6), we find a similar perspective. At the i-level a natural number is treated as a set, whereas at the e-level the numbers can be treated as elements of i. This enables us to use an adapted notion of instant structure as discussed by K&R, thus creating a means to bring about the shift from the naturals to the reals and reversely. I will take advantage of that by transforming Figure 4 so that it can be compared with Figure 2, with i_2 as $\{0, 1, 2\}$ and i_3 as $\{0, 1, 2, 3\}$.[5] It is clear that the two

[5] I deviate from (6) by not including the zero value as a counting value in the predication.

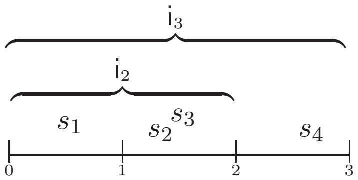

Figure 5: $\mathsf{i}_2 <_{\mathsf{i}} \mathsf{i}_3$

perspectives on numbers play a role in Figure 5.

One difference between Figure 5 and Figure 2 is that Postulate 5 does not apply to the former. To overcome this problem, we replace $\circ$ by $=$. This leads to Definition 4.1, in which the labels of the model structures used by K&R are retained in order to underscore the parallelism with Definition 5.6.1.

Definition 4.1

Let I be the set of indices isomorphic to the set of natural numbers with $\mathcal{E} = \langle \mathsf{I}, \leq \rangle$, obeying the postulates P1 - P7, with $\circ$ replaced by $=$. Then $\mathcal{I}(\mathcal{E}) = \langle \mathsf{I}(\mathcal{E}), <_{\mathsf{i}} \rangle$, with $\mathsf{I}(\mathcal{E})$ the set of instants i as defined in (i) and $<_{\mathsf{i}}$ the precedence relation as defined in (ii):

(i) i is an instant of $\mathcal{E}$ if
  a. $\mathsf{i} \subseteq \mathsf{I}$;
  b. $\mathsf{e}_1, \mathsf{e}_2 \in \mathsf{i} \Rightarrow \mathsf{e}_1 \leq \mathsf{e}_2$

(ii) For all instants $\mathsf{i}_1, \mathsf{i}_2 : \mathsf{i}_1 <_{\mathsf{i}} \mathsf{i}_2$ iff there are $\mathsf{e}_1 \in \mathsf{i}_1, \mathsf{e}_2 \in \mathsf{i}_2$ such that $\mathsf{e}_1 < \mathsf{e}_2$

This produces the instant structure underlying the Path structure described above. From this it should be possible to define a function *Int*: $\mathsf{I}(\mathcal{E}) \longrightarrow \mathsf{Int}$ meeting the requirements of Definition 5.6.2 and embedding Path structure into real time. In Verkuyl 1993 this is done by an actualization function which connects the Reals to Figure 5 in much the same way in which a performance of a sonata is related to the score.[6] At that point we meet events taking place in real time.

5 Comparison and conclusion

Now, this cannot be the whole story. It is evident that Definition 4.1 provides for the right notion of i-precedence, but one could say that it fails when it comes to describing what happens if Judith eating four sandwiches overlaps with Mary's drinking a glass of beer. After all, these things happen. The answer to this problem is to make sure that we do not look at the model structure but rather

This is part of my localistic perspective that does not play a role here. It does not change anything concerning the point at issue.

[6]This is achieved by an inverse rounding-off function.

at the predication about these two events. It is by and through predication that we get at model structure. So, let us consider *Judith ate four sandwiches and Mary drank a glass of beer*. It is obvious that in this case we have two Path structures of the kind illustrated in Figure 5. And it is obvious too, that here we need the overlap-relation as defined e.g. in van Benthem 1983:62 where overlap takes place in real time.

Considering the differences between the event-time strategy and the number-time strategy one can only conclude that Figure 2 is contaminated by the macro-notion of event. An event in the sense of Figure 2 turns out to require complete Path structure but as this is brought about by predication, one cannot assume events to be out there even though many things happen. Events are construed by predication. That is why events are very useful in discourse, because discourse is after all a sequence of predications. The present argument suggests that overlap is necessary only in comparing complete Path structures. We want to locate events as they take place with respect to each other and allow for partial simultaneity. But this can be achieved in the Reals.

Perhaps a metaphor may guide us here: if I read a score, I would not call a sequence of discrete notes to be played, say, by the clarinet an event or a set of events. If I compare the scores for the clarinet player and the hobo player, then in spite of the overlapping structure that I see in comparing the two bars, I would not call the two "Path structures" events. The overlap relation between the two sequences in the score shows up as an overlap between events only during a performance. This is exactly what happens in a discourse, when applied to a certain model. As long as K&R analyze discourse as scores, they had better stay more abstract by using numbers (abstract notes) rather than events (tones). So, my conclusion is that K&R by departing from the model structure side, have made the same sort of mistake as Davidson, in suggesting that a macro-notion can be applied at the micro-level, i.e. at the level at which predication is built up from the information of its parts.

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