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#### Abstract

This paper introduces a semantic framework in which the meaning of a sentence embodies both its *informative* and its *attentive* content. This framework allows for an improved implementation of the analysis of *might* proposed in Ciardelli, Groenendijk, and Roelofsen (2009), which in turn builds on an idea from Groenendijk, Stokhof, and Veltman (1996). The analysis sheds new light on the way in which *might* interacts with conjunction, disjunction, and negation, which is puzzling for the standard modal account of *might*, as well as its treatment in update semantics.

This paper is dedicated to Jeroen Groenendijk, Martin Stokhof, and Frank Veltman, on the occasion of their upcoming retirement, with deepest respect and gratitude.

# 1 Introduction

Our point of departure will be a passage from *Coreference and Modality* (Groenendijk, Stokhof, and Veltman, 1996), one of the most memorable and inspiring papers that I read as a student, which brings together the main ideas from *dynamic predicate logic* (Groenendijk and Stokhof, 1991) and *update semantics* (Veltman, 1996), in particular the dynamic treatment of discourse anaphora ('coreference') and the dynamic treatment of *might* ('modality'). The passage in question puts forth the idea that the core semantic contribution of *might* sentences, or at least one of their core semantic contributions, may be taken to lie in their potential to *draw attention* to certain possibilities. Groenendijk, Stokhof, and Veltman (1996) put it like this:<sup>1</sup>

"In many cases, a sentence of the form  $might-\varphi$  will have the effect that one becomes aware of the possibility of  $\varphi$ ."

It was thought that capturing this aspect of the meaning of *might* would require a more complex notion of possible worlds and information states, and a different way to think about growth of information. Thus, immediately following the above quotation, Groenendijk, Stokhof, and Veltman write that their framework:

"is one in which possible worlds are total objects, and in which growth of information about the world is explicated in terms of elimination of possible worlds. Becoming aware of a possibility cannot be accounted for in a natural fashion in such an eliminative approach. It would amount to extending partial worlds, rather than eliminating total ones. To account for that aspect of the meaning of *might* a constructive approach seems to be called for."

<sup>\*</sup>Sections 2-4 of this paper were part, in preliminary form, of a 2011 manuscript (Roelofsen, 2011c), though there has been one important change, which will be indicated in the text. Section 1 presents a new argument to motivate the overall approach. I am very grateful to Ivano Ciardelli, Jeroen Groenendijk, and Matthijs Westera for extensive discussion of the ideas presented here and many closely related topics (only the new argument in Section 1 will be a surprise for Jeroen). Financial support from the Netherlands Organization for Scientific Research (NWO) is gratefully acknowledged.

<sup>&</sup>lt;sup>1</sup>See also Swanson (2006), Franke and de Jager (2008), Brumwell (2009), and de Jager (2009).

### 1.1 Attentive *might*

Ciardelli, Groenendijk, and Roelofsen (2009) recently challenged this conclusion. Building on earlier work in *inquisitive semantics* (Groenendijk and Roelofsen, 2009; Ciardelli, 2009, among others), they developed a framework in which the meaning of a sentence does not only embody its informative content, but also its inquisitive and attentive content. Possible worlds are still total objects in this framework, and growth of information is still explicated in terms of eliminating worlds. However, growth of information is no longer the only effect that utterances may have. They may also raise new issues or draw attention to certain possibilities, and the latter is indeed construed as the core semantic contribution of *might* sentences.

Ciardelli *et al.* (2009) take the proposition expressed by a sentence  $\varphi$ , denoted  $[\varphi]$ , to be a set of *possibilities*, which in turn are sets of possible worlds. In uttering  $\varphi$ , a speaker is taken (i) to draw attention to all the possibilities in  $[\varphi]$ , (ii) to provide the information that the actual world is located in at least one of these possibilities, i.e., in  $\bigcup[\varphi]$ , and (iii) to request enough information from other conversational participants to locate the actual world inside a specific possibility in  $[\varphi]$ . Thus, the proposition expressed by a sentence captures its attentive, informative, inquisitive content, all in one go.

This notion of meaning is a further refinement of one familiar from basic inquisitive semantics. There, propositions are only intended to capture informative and inquisitive content, not attentive content. Formally, then, they are also more constrained—they are not construed as arbitrary sets of possibilities, but rather as sets of possibilities that are *downward closed* (see, e.g., Ciardelli *et al.*, 2013; Roelofsen, 2013, for justification of this constraint).

Now, the moment we enrich our notion of meaning, an important question that immediately arises is how these enriched meanings are to be *composed*. For instance, given the propositions expressed by two sentences  $\varphi$  and  $\psi$ , how should we determine the propositions expressed by the more complex sentences  $\varphi \lor \psi$ ,  $\varphi \land \psi$ , and  $\neg \varphi$ ?

Ciardelli *et al.* (2009) address this issue and present a recursive semantics for the language of propositional logic, enriched with an operator corresponding to *might*, denoted as usual by  $\diamond$ . This system, which we will refer to as CGR-09, makes a number of interesting predictions. For instance, it straightforwardly accounts for the observation that (6), (7), and (8) below are intuitively all equivalent (Zimmermann, 2000, p.258–259), an observation that is very difficult, if not impossible, to explain for the traditional modal analysis of *might* or the dynamic analysis developed by Veltman (1996) and further pursued by Groenendijk *et al.* (1996).<sup>2</sup>

(1)	John might be in Paris or in London.	$\Diamond (p \lor q)$
(2)	John might be in Paris or he might be in London.	$\Diamond p \lor \Diamond q$
(3)	John might be in Paris and he might be in London.	$\Diamond p \land \Diamond q$

### 1.2 Remaining problems

However, the CGR-09 system faces a number of problems, both at a foundational level and at the level of empirical predictions. The main problem, as I see it, is that it is unclear how the standard algebraic treatment of the connectives (e.g., the treatment of disjunction and conjunction as *join* and *meet* operators, respectively) can be generalized to this richer setting. As long as we restrict ourselves to informative and inquisitive content, leaving attentive content of the picture, such a generalization can be established quite straightforwardly (Roelofsen, 2013). However,

<sup>&</sup>lt;sup>2</sup>Analogous examples with *deontic modals* have also been discussed widely in the literature, as exemplifying the phenomenon of *free choice permission* (Kamp, 1973; Zimmermann, 2000; Geurts, 2005; Simons, 2005; Schulz, 2005; Alonso-Ovalle, 2006; Aloni, 2007; Fox, 2007; Klinedinst, 2007; Chemla, 2009, among many others).

the moment we start taking attentive content into consideration as well, things become more intricate, and, despite some vigorous attempts (e.g., Roelofsen, 2011a,b; Westera, 2012), no fully satisfactory result has been established in this area.

To see what this means more concretely, consider the case of conjunction. In classical propositional logic,  $[\varphi \land \psi]$  is defined as the *meet* of  $[\varphi]$  and  $[\psi]$ . This means that  $\varphi \land \psi$  is characterized as being:

- (i) at least as informative as  $\varphi$  and  $\psi$ , and
- (ii) not any more informative than is necessary to satisfy (i)

In inquisitive semantics,  $[\varphi \land \psi]$  is still defined as the *meet* of  $[\varphi]$  and  $[\psi]$ . In this richer setting, this means that  $\varphi \land \psi$  is characterized as being:

- (i) at least as informative and inquisitive as  $\varphi$  and  $\psi$ , and
- (ii) not any more informative or inquisitive than is necessary to satisfy (i)

Now, when taking attentive content into consideration, we would like to preserve the essence of this treatment of conjunction. Thus, in this setting we would like to characterize  $\varphi \wedge \psi$  as being:

- (i) at least as informative, inquisitive, and attentive as  $\varphi$  and  $\psi$ , and
- (ii) not any more informative, inquisitive, or attentive than necessary to satisfy (i)

The problem is that we don't know how to define in an appropriate way when one sentence is at least as informative, inquisitive, and attentive as another. In other words, it is not clear how the notion of *entailment* is to be defined in this setting. And as long as this remains unsettled, the algebraic treatment of the connectives does not get off the ground.

For lack of a more principled approach, then, CGR-09 has to resort to an *ad hoc* treatment of conjunction in terms of pointwise intersection:

$$[\varphi \land \psi] \coloneqq \{\alpha \cap \beta \mid \alpha \in [\varphi] \text{ and } \beta \in [\psi]\}$$

Of course, this is still rather natural, especially since the *meet* operation in classical logic (and also in inquisitive semantics) amounts to intersection. Going from intersection to pointwise intersection seems to be a natural step. However, it has a number of problematic consequences. For instance, conjunction is no longer *idempotent* under this treatment. That is,  $\varphi \land \varphi$  is no longer always equivalent with  $\varphi$  itself. Another, more concrete and specific problem, noted by Luis Alonso-Ovalle (p.c.), is that a conjunction of two *might* sentences,  $\Diamond p \land \Diamond q$ , comes to express exactly the same proposition as  $\Diamond (p \lor q \lor (p \land q))$ . If we consider corresponding sentences in English we find clear differences:

- (4) ✓ John might speak English and he might speak French, but of course he doesn't speak both.
- (5) #John might speak English or French, or both, but of course he doesn't speak both.

These considerations reveal that there is still a very fundamental issue left open by CGR-09, which results in concrete empirical problems as well.

### 1.3 Hidden assumptions

What is the source of these problems, and how could they be overcome? My diagnosis is that the CGR-09 system tries to do too much at once. On the one hand, it keeps the formal objects that model the meaning of a sentence relatively uninvolved, i.e., propositions are simply defined as sets of possibilities. On the other hand, it places a heavy burden on these objects, namely, to capture not only informative content, but also inquisitive and attentive content. We saw how this is done: in uttering  $\varphi$ , a speaker is taken to (i) draw attention to all the possibilities in  $[\varphi]$ , (ii) provide the information that the actual world is located in at least one of these possibilities, i.e., in  $\bigcup[\varphi]$ , and (iii) request enough information from other conversational participants to locate the actual world inside a specific possibility in  $[\varphi]$ . At first sight, this seems to work out: even though the internal structure of propositions is kept relatively simple, they seem sufficiently fine-grained to capture these three types of content.

However, the approach involves some implicit assumptions that have not received our full attention yet, and which may well be the source of the problems that have been encountered. Namely, it is implicitly assumed that the informative, inquisitive, and attentive content of a sentence are related to one another in a particular way. To see what this means more concretely, let us make more explicit what we mean by informative, inquisitive, and attentive content. First, in uttering a sentence  $\varphi$ , a speaker is taken to provide the information that the actual world is located in  $\bigcup[\varphi]$ . Thus, the informative content of  $\varphi$ , info $(\varphi)$ , can be defined as  $\bigcup[\varphi]$ . Second, a speaker is taken to request enough information from other conversational participants to locate the actual world inside a specific possibility in  $[\varphi]$ . The inquisitive content of  $\varphi$ , then, can be identified with the set of all pieces of information that would satisfy this request. A piece of information can be modeled as a set of possible worlds, and it satisfies the request that is made in uttering  $\varphi$  just in case it is contained in one of the possibilities in  $[\varphi]$ . Thus, the inquisitive content of  $\varphi$ , inq $(\varphi)$ , can be defined as  $[\varphi]^{\downarrow} \coloneqq \{\alpha \mid \alpha \subseteq \beta \text{ for some } \beta \in [\varphi]\}$ , the downward closure of  $[\varphi]$ . Finally, a speaker is taken to draw attention to all the possibilities in  $[\varphi]$ . Thus, the attentive content of  $\varphi$ , att $(\varphi)$ , can simply be defined as  $[\varphi]$ . Summing up:

Definition 1 (Informative, inquisitive, and attentive content in CGR-09).

- $info(\varphi) \coloneqq \bigcup [\varphi]$
- $\operatorname{inq}(\varphi) \coloneqq [\varphi]^{\downarrow}$
- att( $\varphi$ ) := [ $\varphi$ ]

Now, it is clear that  $info(\varphi)$ ,  $inq(\varphi)$ , and  $att(\varphi)$  are related to one another in a particular way in CGR-09. In particular, we always have that:

Fact 1 (Relations between different types of content in CGR-09).

- 1.  $\operatorname{info}(\varphi) = \bigcup \operatorname{inq}(\varphi)$
- 2.  $info(\varphi) = \bigcup att(\varphi)$
- 3.  $inq(\varphi) = att(\varphi)^{\downarrow}$
- 4.  $\operatorname{att}(\varphi) \subseteq \operatorname{inq}(\varphi)$

The first equivalence,  $info(\varphi) = \bigcup inq(\varphi)$ , is inherited from work on basic inquisitive semantics, and has been amply motivated there (see, e.g., Ciardelli *et al.*, 2012). However, the other items, which all involve attentive content, have never been motivated explicitly. Item 2,  $info(\varphi) = 0$ 

 $\bigcup$  att( $\varphi$ ), is justified under the assumption that by drawing attention to a certain possibility, a speaker indicates that the actual world may be located inside this possibility, and moreover, that by not drawing attention to any possibility containing a certain possible world w, a speaker discards w as a candidate for the actual world. However, items 3 and 4, which concern the relation between inquisitive and attentive content, cannot be motivated so straightforwardly. In fact, I think that these items cannot be justified at all.

To see this, first let us paraphrase what these items say. Item 4 says that every possibility that a speaker draws attention to in uttering  $\varphi$  must coincide with a piece of information that satisfies the request for information that the speaker issues in uttering  $\varphi$ . Item 3 further strengthens this requirement: namely, it says that the pieces of information that satisfy the request for information that the speaker issues in uttering  $\varphi$  are precisely those that are contained in one of the possibilities that the speaker draws attention to.

There are cases in which these requirements seem too strong. Consider, for instance, a conjunction of two polar questions,  $?p \land ?q$ . Intuitively, in uttering this sentence, a speaker draws attention to the possibility that p, i.e., the possibility consisting of all worlds where p is true. However, providing the information that p is true is not sufficient to satisfy the request for information that is issued in uttering  $?p \land ?q$ . After all, it does not say anything about q.

But the moment we agree on these two basic intuitions, i.e., (i) that a speaker in uttering  $?p \wedge ?q$  draws attention to the possibility that p, and (ii) that providing the information that p is true is not sufficient to satisfy the request that is issued in uttering  $?p \wedge ?q$ , then we are forced to conclude that items 3 and 4 above are not warranted, and therefore, that the integrated way in which informative, inquisitive, and attentive content are modeled in CGR-09 is, unfortunately, on the wrong track.

## 1.4 Bare bones

The aim of this paper is to take the central idea from CGR-09, which in turn has its roots in the quoted passage from *Coreference and Modality*, and implement it in a more minimalistic framework, namely, one in which propositions capture informative and attentive content in an integrated way, but not inquisitive content. This seems a reasonable step to take, since the envisioned treatment of *might*, and the main phenomena that it aspires to account for, do not seem to rely in any essential way on taking inquisitive aspects of meaning into consideration.

The analysis will be developed in three stages. First, in Section 2, we define an attentive semantics for the language of propositional logic, providing a detailed justification for its treatment of the connectives. Then, in Section 3, we show how this framework can be used to characterize the semantics of attentive *might*. Finally, in Section 4, we discuss the pragmatic perspective that our attentive semantics gives rise to, which is richer than the standard Gricean perspective, and we explore the consequences of this refinement for the interpretation of *might* sentences. Section 5 concludes.

# 2 Information and attention

We take propositions to be sets of possibilities, where each possibility in turn is a non-empty set of possible worlds. In uttering a sentence  $\varphi$ , a speaker is taken to:

1. Draw attention to all the possibilities in  $[\varphi]$  as possibilities that may contain the actual world

Floris Roelofsen

2. Provide the information that the actual world must be contained in at least one of the possibilities in  $[\varphi]$ 

Thus, propositions capture both informative and attentive content, but, unlike in CGR-09, they are not intended to capture inquisitive content. Note that possibilities are defined as *non-empty* sets of possible worlds. This is because it would be incoherent for a speaker to draw attention to the possibility that the actual world may be contained in  $\emptyset$ .<sup>3</sup>

In the remainder of this section, we will develop a concrete semantics for the language of propositional logic that is based on this general conception of propositions.

### 2.1 Attentive propositional logic

Let  $\mathcal{P}$  be a set of atomic sentences, and let  $\mathcal{L}_{\mathcal{P}}$  be the set of sentences that are built up from the elements of  $\mathcal{P}$  using the Boolean connectives,  $\land$ ,  $\lor$ , and  $\neg$  in the usual way.<sup>4</sup>

Definition 2 (Possible worlds, possibilities, and propositions).

- A possible world is a function from  $\mathcal{P}$  to  $\{0,1\}$
- A possibility is a non-empty set of possible worlds
- A proposition is a set of possibilities

The proposition expressed by a sentence  $\varphi$  will be denoted by  $[\varphi]$ , and the possibilities in  $[\varphi]$  will be referred to as the possibilities for  $\varphi$ . The set of all possible worlds will be denoted by  $\omega$ , the set of all possibilities by  $\Omega$ , and the set of all propositions by  $\Sigma$ .

For any proposition A, we will refer to  $\bigcup A$  as the *informative content* of A, and derivitavely, for any sentence  $\varphi$ , we will refer to  $\bigcup [\varphi]$  as the informative content of  $\varphi$ .

**Definition 3** (Informative content). For any  $A \in \Sigma$  and every  $\varphi \in \mathcal{L}_{\mathcal{P}}$ :

- $info(A) \coloneqq \bigcup A$
- $info(\varphi) \coloneqq \bigcup [\varphi]$

For any proposition A and any set of worlds  $\beta$ ,  $A_{\beta}$  denotes the *restriction* of A to  $\beta$ , which is the set of all possibilities that can be obtained by intersecting  $\beta$  with some possibility  $\alpha \in A$ .

**Definition 4** (Restricting propositions).  $A_{\beta} := \{ \alpha \cap \beta \in \Omega \mid \alpha \in A \}$ 

We are now ready to state the recursive semantics for  $\mathcal{L}_{\mathcal{P}}$ .

**Definition 5** (Semantics for  $\mathcal{L}_{\mathcal{P}}$ ).

- 1.  $[p] := \{ \{ w \mid w(p) = 1 \} \}$  if p is atomic
- 2.  $[\neg \varphi] := \{ \overline{\mathsf{info}(\varphi)} \}$  if  $\overline{\mathsf{info}(\varphi)} \neq \emptyset$ , otherwise  $[\neg \varphi] := \emptyset$
- 3.  $[\varphi \lor \psi] \coloneqq [\varphi] \cup [\psi]$

<sup>&</sup>lt;sup>3</sup>This is where we diverge from Roelofsen (2011c), where possibilities were defined as arbitrary sets of possible worlds, and propositions as non-empty sets of possibilities. Apart from being conceptually more attractive, the current setup also yields a better behaved notion of entailment. See Footnote 5 for illustration of this point. I am very grateful to Matthijs Westera for suggesting this change and making me aware of its benefits.

<sup>&</sup>lt;sup>4</sup>We do not include implication as a connective in our basic language, because it involves certain complexities that are orthogonal to the main issues addressed here.

Floris Roelofsen

4. 
$$[\varphi \land \psi] \coloneqq [\varphi]_{info(\psi)} \cup [\psi]_{info(\varphi)}$$

We will refer to this system as *attentive propositional logic*, APL. We will briefly go through the clauses one by one. In doing so we will speak of sentences as 'providing information' and 'drawing attention to possibilities'. Strictly speaking, sentences themselves of course do not provide information or draw attention to possibilities. Rather, this is done by *speakers* in uttering these sentences. However, the explanation of the semantics will be more illuminating if we allow ourselves to be somewhat sloppy in this respect.

Atoms. The atomic clause says that an atomic sentence p draws attention to a single possibility, namely the possibility that consists of all worlds where p is true. It thereby provides the information that the actual world must be one where p is true.

**Negation.** The clause for negation says that a negated sentence  $\neg \varphi$  draws attention to at most one possibility, which is the complement of  $info(\varphi)$ . If this complement is empty, then  $\neg \varphi$  does not draw attention to any possibility. In any case,  $\neg \varphi$  provides the information that the actual world is not contained in  $info(\varphi)$ .

**Disjunction.** The clause for disjunction says that  $\varphi \lor \psi$  draws attention to all the possibilities that  $\varphi$  draws attention to, plus all the possibilities that  $\psi$  draws attention to. This means that  $\varphi \lor \psi$  provides the information that the actual world is included in at least one of the possibilities that either  $\varphi$  or  $\psi$  draws attention to. Thus,  $\varphi \lor \psi$  provides the information that the actual world is contained in info $(\varphi) \cup info(\psi)$ .

**Conjunction.** The clause for conjunction says that  $\varphi \wedge \psi$  draws attention to all the possibilities for  $\varphi$  restricted to  $info(\psi)$ , and to all the possibilities for  $\psi$  restricted to  $info(\varphi)$ . This means that it provides the information that the actual world is contained in  $info(\varphi) \cap info(\psi)$ . It may be helpful to note that the clause for conjunction may also be formulated, equivalently, as follows:

4' 
$$[\varphi \land \psi] \coloneqq ([\varphi] \cup [\psi])_{info(\varphi) \cap info(\psi)}$$

So, to compute the proposition expressed by  $\varphi \wedge \psi$  we may simply collect all the possibilities for  $\varphi$  and all the possibilities for  $\psi$ , and then restrict all these possibilities to  $info(\varphi) \cap info(\psi)$ , i.e., the set of worlds that are compatible with the information that  $\varphi$  and  $\psi$  together provide.

# 2.2 Comparison with classical propositional logic

Let us briefly compare APL with classical propositional logic, CPL. For any sentence  $\varphi \in \mathcal{L}_{\mathcal{P}}$ , let  $|\varphi|$  denote the proposition expressed by  $\varphi$  in CPL. APL is of course richer than CPL, because it captures both informative and attentive content. However, as far as informative content is concerned, APL coincides with CPL. That is, for every  $\varphi \in \mathcal{L}_{\mathcal{P}}$ , we have that  $info(\varphi) = |\varphi|$ . In this sense, APL is a conservative extension of CPL.

**Fact 2** (APL and CPL). For every  $\varphi \in \mathcal{L}_{\mathcal{P}}$ : info $(\varphi) = |\varphi|$ 

Note that, since  $[\neg\varphi]$  is defined as  $\{\overline{\bigcup[\varphi]}\}\)$ , we always have that  $[\neg\neg\varphi] = \{\bigcup[\varphi]\} = \{\inf o(\varphi)\} = \{|\varphi|\}\)$ . So by taking the double negation of a sentence  $\varphi$  we always get a sentence that expresses a proposition consisting of a single possibility, which coincides with the classical meaning of  $\varphi$ .

**Fact 3** (Double negation). For every  $\varphi \in \mathcal{L}_{\mathcal{P}}$ :  $[\neg \neg \varphi] = \{ |\varphi| \}$ 

Floris Roelofsen

Finally, note that our language is *functionally complete*, in the sense that for every proposition it is possible to find a sentence that expresses that proposition.

Fact 4 (Functional completeness).

For every proposition  $A \in \Sigma$ , there is a sentence  $\varphi \in \mathcal{L}_{\mathcal{P}}$  such that  $[\varphi] = A$ .

**Proof.** Recall that  $\mathcal{P}$  is assumed to be finite. This means that for every world w, there is a sentence  $\varphi_w$  such that  $[\varphi_w] = \{\{w\}\}$ , namely:

$$\varphi_w = \bigwedge \{ p \mid w(p) = 1 \} \land \bigwedge \{ \neg p \mid w(p) = 0 \}$$

But then for every possibility  $\alpha$ , there is a sentence  $\varphi_{\alpha}$  such that  $[\varphi_{\alpha}] = \{\alpha\}$ , namely  $\neg \neg \lor \{\varphi_{w} \mid w \in \alpha\}$ . And this means that for every proposition A, there is a sentence  $\varphi_{A}$  such that  $[\varphi_{A}] = A$ , namely  $\lor \{\varphi_{\alpha} \mid \alpha \in A\}$ .  $\Box$ 

# 2.3 Entailment, homogeneity, and refinement

In CPL, one sentence entails another just in case the former is at least as informative as the latter. Since the meaning of a sentence in CPL is identified with its informative content, entailment can simply be defined as meaning inclusion in this setting:

$$\varphi \models_{\mathsf{CPL}} \psi \text{ iff } |\varphi| \subseteq |\psi|$$

In APL, sentences can be ordered in terms of their informative content, but also in terms of their attentive content. As in the classical setting,  $\varphi$  is at least as informative as  $\psi$  iff  $info(\varphi) \subseteq info(\psi)$ . As for attentiveness, it is natural to say that  $\varphi$  is at least as attentive as  $\psi$  iff  $\varphi$  draws attention to all the possibilities that  $\psi$  draws attention to, to the extent that they are compatible with the informative content of  $\varphi$ .

Definition 6 (Informative and attentive orders).

- $\varphi \geq_{info} \psi$  iff  $info(\varphi) \subseteq info(\psi)$
- $\varphi \geq_{\mathsf{att}} \psi$  iff  $[\psi]_{\mathsf{info}(\varphi)} \subseteq [\varphi]$

These orders can be combined in several ways. In particular, we will say that  $\varphi$  entails  $\psi$ ,  $\varphi \vDash \psi$ , iff  $\varphi$  is at least as informative and at least as attentive as  $\psi$ ,  $\varphi \ge_{info} \psi$  and  $\varphi \ge_{att} \psi$ .<sup>5</sup> Besides entailment, we will also introduce a notion of *homogeneity*:  $\varphi$  is at least as homogeneous as  $\psi$ ,  $\varphi \ltimes \psi$  iff  $\varphi$  is at least as informative and at *most* as attentive as  $\psi$ ,  $\varphi \ge_{info} \psi$  and  $\varphi \le_{att} \psi$ .<sup>6</sup> Thus, one sentence is more homogeneous than another if it (i) leaves fewer possible candidates for the actual world, and (ii) draws attention to fewer different possibilities.

Definition 7 (Entailment and homogeneity).

<sup>&</sup>lt;sup>5</sup>Recall from Footnote 3 that in Roelofsen (2011c) possibilities were defined as arbitrary (possibly non-empty) sets of possible worlds and propositions as non-empty sets of possibilities. With these definitions of possibilities and propositions, the notion of entailment gives slightly different results than it does in the present setting. For instance, we would get that  $p \neq p \lor \neg p$ . These kind of results are undesirable, especially in view of the crucial role that attentive entailment may be taken to play in the formal characterization of conversational relevance, as proposed in Westera (2013).

<sup>&</sup>lt;sup>6</sup>A parallel notion of homogeneity exists in inquisitive semantics (see, e.g., Groenendijk and Roelofsen, 2009). There,  $\varphi$  is defined to be at least as homogeneous as  $\psi$  iff  $\varphi$  is at least as informative and at most as inquisitive as  $\psi$ .

Floris Roelofsen

- $\varphi \vDash \psi$  iff  $\varphi \ge_{info} \psi$  and  $\varphi \ge_{att} \psi$
- $\varphi \ltimes \psi$  iff  $\varphi \geq_{info} \psi$  and  $\varphi \leq_{att} \psi$

It turns out that in APL, meaning inclusion does not correspond to  $\vDash$  but rather to  $\ltimes$ .

Fact 5 (Homogeneity amounts to meaning inclusion).

•  $\varphi \ltimes \psi$  iff  $[\varphi] \subseteq [\psi]$ 

**Proof.** Suppose that  $\varphi \ltimes \psi$ , and suppose that  $\alpha \in [\varphi]$ . Then, since  $\varphi \ge_{info} \psi$ , we have that  $\alpha \subseteq info(\psi)$ . But then, since  $\psi \ge_{att} \varphi$ , we must have that  $\alpha \in [\psi]$ . So  $[\varphi] \subseteq [\psi]$ . Vice versa, if  $[\varphi] \subseteq [\psi]$ , it immediately follows that  $\varphi \ge_{info} \psi$  and  $\psi \ge_{att} \varphi$ .  $\Box$ 

Note that entailment and homogeneity are defined above as relations between sentences. It will be useful to define them as relations between *propositions* as well.

**Definition 8** (Propositional entailment and homogeneity). Let A and B be two propositions. Then:

- $A \ge_{info} B$  iff  $info(A) \subseteq info(B)$
- $A \geq_{\mathsf{att}} B$  iff  $B_{\mathsf{info}(A)} \subseteq A$
- $A \vDash B$  iff  $A \ge_{info} B$  and  $A \ge_{att} B$
- $A \ltimes B$  iff  $A \ge_{info} B$  and  $A \le_{att} B$

Evidently, there is a straightforward correspondence between the sentential notions of entailment and homogeneity, and the propositional notions.

Fact 6 (Sentential and propositional entailment and homogeneity).

- $\varphi \vDash \psi$  iff  $[\varphi] \vDash [\psi]$
- $\varphi \ltimes \psi$  iff  $[\varphi] \ltimes [\psi]$  iff  $[\varphi] \subseteq [\psi]$

However, some properties of entailment and homogeneity hold only at the level of propositions. For instance, as desired, entailment and homogeneity form *partial orders* on the set of all propositions  $\Sigma$ .<sup>7</sup>

**Fact 7** (Partial orders).  $\models$  and  $\ltimes$  form partial orders on  $\Sigma$ .

<sup>&</sup>lt;sup>7</sup>Entailment and homogeneity do not form partial orders on the set of all sentences, since two different sentences may very well express exactly the same proposition and therefore be just as informative and just as attentive. This means that entailment and homogeneity, conceived of as relations between sentences, are not anti-symmetric.

Floris Roelofsen

**Proof.** The claim for  $\ltimes$  immediately follows from the fact that it amounts to meaning inclusion. As for entailment, we have to show that  $\vDash$  is reflexive, transitive, and anti-symmetric. It is clear that  $\vDash$  is reflexive. For transitivity, suppose that  $A \vDash B$  and  $B \vDash C$ . Then clearly  $A \ge_{info} C$ . It remains to be shown that  $A \ge_{att} C$ . Let  $\gamma \in C$  and  $\gamma \cap info(A) \neq \emptyset$ . We have to show that  $\gamma \cap info(A) \in A$ . First, since  $info(A) \subseteq info(B)$ , we have that  $\gamma \cap info(B) \neq \emptyset$ . But then, since  $B \ge_{att} C$ , we must have that  $\gamma \cap info(B) \in B$ . Now consider  $\gamma \cap info(B) \cap info(A)$ . Since  $info(A) \subseteq info(B)$ , this just amounts to  $\gamma \cap info(A)$ , which we already know is non-empty. But then, since  $A \ge_{att} B$ , it must be included in A. Thus, indeed,  $\gamma \cap info(A) \in A$ .

Finally, to establish that  $\vDash$  is anti-symmetric, assume that  $A \vDash B$  and  $B \vDash A$ . We have to show that A = B. Let  $\alpha \in A$ . Since  $info(A) \subseteq info(B)$ , we have that  $\alpha \cap info(B)$  is non-empty and actually just amounts to  $\alpha$  itself. But then, since  $B \ge_{att} A$ , we must have that  $\alpha \in B$ . This means that  $A \subseteq B$ , and in the same way we can establish that  $B \subseteq A$ . Thus, indeed, A = B.  $\Box$ 

There is one proposition, namely  $\emptyset$ , which entails every other proposition. We will therefore refer to  $\emptyset$  as the *absurd* proposition, and to sentences that express  $\emptyset$  as *contradictions*. An example of a contradiction is the sentence  $p \wedge \neg p$ . We will use  $\bot$  as an abbreviation of this sentence.

There is no proposition that is entailed by all other propositions. This means that *tautologies* cannot be defined in terms of entailment in the usual way, i.e., as sentences that are entailed by all other sentences. Instead, we will call a sentence a tautology just in case every other sentence is a *refinement* of it in the following sense.

### **Definition 9** (Refinement).

 $\varphi$  is a refinement of  $\psi$ ,  $\varphi \geq \psi$ , if and only if:

- 1.  $\varphi \geq_{info} \psi$  and
- 2. for all  $\beta \in [\psi]$  there is a  $C \subseteq [\varphi]$  such that  $\beta \cap info(\varphi) = \bigcup C$

In order for  $\varphi$  to be a refinement of  $\psi$ , first of all  $\varphi$  must be at least as informative as  $\psi$ . However,  $\varphi$  does not have to draw attention to every possibility that  $\psi$  draws attention to (restricted to  $\inf(\varphi)$ ). Rather, for every possibility  $\beta$  that  $\psi$  draws attention to, there has to be a set of possibilities  $C \subseteq [\varphi]$  such that  $\beta \cap \inf(\varphi)$  coincides with  $\bigcup C$ . Intuitively, C can be thought of as a *cover* of  $\beta \cap \inf(\varphi)$ . Again this notion of refinement can also be defined at the level of propositions (as opposed to sentences).

### **Definition 10** (Propositional refinement).

 $A \succeq B$  if and only if:

- 1.  $A \geq_{info} B$  and
- 2. for all  $\beta \in B$  there is a  $C \subseteq A$  such that  $\beta \cap \bigcup A = \bigcup C$

Clearly, there is a straightforward correspondence between sentential and propositional refinement.

Fact 8 (Sentential and propositional refinement).

•  $\varphi \succeq \psi$  iff  $[\varphi] \succeq [\psi]$ 

Floris Roelofsen



Figure 1: Two propositions showing that refinement is not anti-symmetric.

There is one proposition, namely  $\{\omega\}$ , which has the special property that every other proposition is a refinement of it. We will therefore refer to  $\{\omega\}$  as the *trivial* proposition, and to sentences that express this trivial proposition as *tautologies*. An example of a tautology is the sentence  $\neg \neg (p \lor \neg p)$ . We will use  $\top$  as an abbreviation of this sentence.

Refinement is strictly weaker than entailment.

Fact 9 (Refinement and entailment).

- 1. For every A and B: if  $A \vDash B$  then also  $A \succeq B$
- 2. There are A and B such that  $A \ge B$  but  $A \notin B$

**Proof.** The first claim follows directly from the definitions. For the second claim, take A to consist of a single possibility  $\alpha$ , and take B to consist of two mutually exclusive possibilities which are both contained in  $\alpha$ . Then B is a refinement of A, but it does not entail A, because it does not draw attention to  $\alpha$  restricted to info(B).

Unlike entailment and homogeneity, the refinement relation does *not* form a partial order on the set of all propositions.

**Fact 10** (No partial order).  $\geq$  does not form a partial order on  $\Sigma$ .

**Proof.**  $\geq$  is clearly reflexive and transitive, but it is not anti-symmetric. To see this, consider the following two propositions:

- $A = [\top \lor (p \land q) \lor (p \land \neg q)]$
- $B = [\top \lor (p \land q) \lor (p \land \neg q) \lor p]$

These propositions are depicted in figures 1(a) and 1(b), respectively. In these figures, 11 is a world where both p and q are true, 10 is a world where p is true and q is false, etcetera. From inspecting the figures, it will be clear that  $A \ge B$  and  $B \ge A$ , but  $A \ne B$ . So  $\ge$  is not anti-symmetric.

### 2.4 Algebraic characterization of APL

Given the notions of entailment, homogeneity, and refinement, the semantic behavior of the connectives in APL can be characterized in algebraic terms. Recall that in CPL, conjunction behaves semantically as a *meet* operator, disjunction as a *join* operator, and negation as a

*complementation* operator w.r.t. entailment. We will show that in APL, conjunction behaves again as a *meet* operator w.r.t.  $\vDash$ , while disjunction behaves as a *join* operator w.r.t.  $\ltimes$ , and negation behaves as a *pseudo-complementation* operator w.r.t.  $\vDash$  and  $\geq$ . Let us first provide definitions of these algebraic notions.

Definition 11 (Meets, joins, and pseudo-complements).

Let A and B be two propositions. Then:

- The meet of A and B w.r.t.  $\vDash$ , if it exists, is the unique proposition M such that:
  - 1.  $M \vDash A$  and  $M \vDash B$
  - 2. For every proposition P, if  $P \vDash A$  and  $P \vDash B$ , then  $P \vDash M$ .

In other words, M is the greatest lower bound of A and B w.r.t.  $\models$ .

- The *join* of A and B w.r.t.  $\ltimes$ , if it exists, is the unique proposition J such that:
  - 1.  $A \ltimes J$  and  $B \ltimes J$
  - 2. For every proposition P, if  $A \ltimes P$  and  $B \ltimes P$ , then  $J \ltimes P$ .

In other words, J is the *least upper bound* of A and B w.r.t.  $\ltimes$ .

- The *pseudo-complement*<sup>8</sup> of A w.r.t.  $\vDash$  and  $\succeq$ , if it exists, is the unique proposition C such that:
  - 1. The meet of A and C w.r.t.  $\vDash$  is the absurd proposition,  $\emptyset$ .
  - 2. For every proposition P that satisfies 1. we have that  $P \geq C$ .

In other words, C is the least refined proposition such that the meet of A and C is the absurd proposition.

Now let us show that the semantic behavior of the connectives in APL can be characterized in terms of these algebraic notions.

**Fact 11** (Conjunction is meet w.r.t.  $\models$ ). For every  $\varphi$  and  $\psi$ ,  $[\varphi \land \psi]$  is the meet of  $[\varphi]$  and  $[\psi]$  w.r.t.  $\models$ .

**Proof.** It follows immediately from the definition of  $[\varphi \land \psi]$  that  $[\varphi \land \psi] \models [\varphi]$  and  $[\varphi \land \psi] \models [\psi]$ . Now suppose that  $[\xi]$  is another proposition that entails both  $[\varphi]$  and  $[\psi]$ . Then we have to show that  $[\xi]$  also entails  $[\varphi \land \psi]$ . First, since  $[\xi] \models [\varphi]$  and  $[\xi] \models [\psi]$ , we have that  $\inf_{0}(\xi) \subseteq \inf_{0}(\varphi)$  and  $\inf_{0}(\xi) \subseteq \inf_{0}(\psi)$ . So  $\inf_{0}(\xi) \subseteq \inf_{0}(\varphi) \cap \inf_{0}(\psi)$ . But  $\inf_{0}(\varphi) \cap \inf_{0}(\psi)$  coincides with  $\inf_{0}(\varphi \land \psi)$ . So  $\inf_{0}(\xi) \subseteq \inf_{0}(\varphi \land \psi)$ . In other words,  $[\xi] \ge_{inf_{0}} [\varphi \land \psi]$ . It remains to be shown that  $[\xi] \ge_{att} [\varphi \land \psi]$ . Let  $\gamma$  be a possibility in  $[\varphi \land \psi]$  such that  $\gamma \cap \inf_{0}(\xi) \neq \emptyset$ . We have to show that  $\gamma \cap \inf_{0}(\xi) \in [\xi]$ . Since  $\gamma$  is in  $[\varphi \land \psi]$ , it is either the intersection of some possibility  $\alpha \in [\varphi]$  with  $\inf_{0}(\psi)$ , or the intersection of some possibility  $\beta \in [\psi]$  with  $\inf_{0}(\varphi)$ . Suppose that it is the intersection of

<sup>&</sup>lt;sup>8</sup>The notion of a pseudo-complement is a standard notion from the algebra that underly intuitionistic logic, which is called Heyting algebra. The notion we define here is non-standard because it is defined in terms of *two* relations, entailment and refinement. The standard notion is defined just in terms of entailment. In all other respects, the definition is the same. It should also be noted that Heyting algebras do not only come with an absolute notion of pseudo-complements, as the one we employ here, but also with a relative notion, which is associated with implication. I don't know at this point whether a suitable notion of relative pseudo-complements can be defined in APL as well. I suspect not.

Floris Roelofsen

some possibility  $\alpha \in [\varphi]$  with  $info(\psi)$ . Then, since  $\gamma \subseteq \alpha$  and  $\gamma \cap info(\xi) \neq \emptyset$ , we also have that  $\alpha \cap info(\xi) \neq \emptyset$ . But then, since  $[\xi] \ge_{att} [\varphi]$ ,  $\alpha \cap info(\xi)$  must be in  $[\xi]$ . Now since  $info(\xi) \subseteq info(\psi)$ , we have that  $\alpha \cap info(\xi) = \alpha \cap info(\psi) \cap info(\xi) = \gamma \cap info(\xi)$ . So  $\gamma \cap info(\xi)$  is in  $[\xi]$ , which is exactly what we set out to show. If we assume that  $\gamma$  is the intersection of some possibility  $\beta \in [\psi]$  with  $info(\varphi)$ , we can show in a similar way that  $\gamma \cap info(\xi)$  is in  $[\xi]$ . So we may conclude that  $[\xi] \ge_{att} [\varphi \land \psi]$ .

Fact 12 (Disjunction is join w.r.t.  $\ltimes).$ 

For every  $\varphi$  and  $\psi$ ,  $[\varphi \lor \psi]$  is the join of  $[\varphi]$  and  $[\psi]$  w.r.t.  $\ltimes$ .

**Proof.** This follows immediately from the fact that  $\ltimes$  amounts to  $\subseteq$ .

**Fact 13** (Negation is pseudo-complement w.r.t.  $\models$  and  $\geq$ ). For every  $\varphi$ ,  $[\neg \varphi]$  is the pseudo-complement of  $[\varphi]$  w.r.t.  $\models$  and  $\geq$ .

**Proof.** First we show that the meet of  $[\varphi]$  and  $[\neg\varphi]$  is the absurd proposition,  $\emptyset$ . We already know that the meet of  $[\varphi]$  and  $[\neg\varphi]$  w.r.t.  $\vDash$  is  $[\varphi \land \neg\varphi]$ . Now suppose that  $\gamma$  is a possibility in  $[\varphi \land \neg\varphi]$ . Then  $\gamma$  must either be the non-empty intersection of some possibility  $\alpha \in [\varphi]$  with info $(\neg\varphi)$ , or the non-empty intersection of some possibility  $\beta \in [\neg\varphi]$  with info $(\varphi)$ . However, this cannot be, since any possibility  $\alpha \in [\varphi]$  is disjoint with info $(\neg\varphi)$  and any possibility  $\beta \in [\neg\varphi]$  is disjoint with info $(\varphi)$ . So  $[\varphi \land \neg\varphi] = \emptyset$ .

Now let  $[\xi]$  be another proposition such that the meet of  $[\varphi]$  and  $[\xi]$  w.r.t.  $\models$  is  $\emptyset$ . Then we have to show that  $[\xi] \ge [\neg \varphi]$ . First, since the meet of  $[\varphi]$  and  $[\xi]$  w.r.t.  $\models$  is  $\emptyset$ , info $(\xi)$  and info $(\varphi)$  must be disjoint. But this means that info $(\xi) \subseteq$  info $(\neg \varphi)$ . So  $[\xi] \ge_{info} [\neg \varphi]$ . It remains to be shown that for every possibility  $\beta \in [\neg \varphi]$  there is a set of possibilities  $C \subseteq [\xi]$  such that  $\alpha \cap info(\xi) = C$ . If  $[\neg \varphi] = \emptyset$  then this is trivially true. On the other hand, if  $[\neg \varphi]$  is not empty, then it consists of a single possibility,  $info(\neg \varphi)$ . But then, since  $info(\xi) \subseteq info(\neg \varphi)$ , the restriction of  $info(\neg \varphi)$  to  $info(\xi)$  is bound to coincide with the union of all the possibilities in  $[\xi]$ . So, indeed,  $[\xi] \ge [\neg \varphi]$ .

Finally, we have to show that  $[\neg\varphi]$  is the *unique* proposition C such that (i) the meet of  $[\varphi]$  and C w.r.t.  $\vDash$  is  $\emptyset$ , and (ii) for every proposition  $[\xi]$  that satisfies (i) we have that  $[\xi] \ge C$ . Since  $\ge$  is not anti-symmetric, it is possible, in principle, that  $[\neg\varphi]$  is not the only proposition satisfying these conditions. Towards a contradiction, let  $[\chi]$  be another such proposition. Then, since both  $[\neg\varphi]$  and  $[\chi]$  satisfy condition (ii) it must be the case that  $[\neg\varphi] \ge [\chi]$  and  $[\chi] \ge [\neg\varphi]$ . From this it immediately follows that  $\inf((\gamma)) = \inf((\neg\varphi))$  and that  $[\chi]$  cannot contain any possibility that is strictly contained in  $\inf((\neg\varphi))$ . But this means that, after all,  $[\chi] = [\neg\varphi]$ . So  $[\neg\varphi]$  is the unique proposition satisfying conditions (i) and (ii), and therefore it is indeed the pseudo-complement of  $[\varphi]$ .

Now let us take a step back, and spell out what these results tell us about the semantic behavior of the connectives in APL.

First, Fact 11 tells us that for any  $\varphi$  and  $\psi$ ,  $[\varphi \land \psi]$  is the weakest proposition that entails both  $[\varphi]$  and  $[\psi]$ . In other words,  $[\varphi \land \psi]$  is the unique proposition with the following properties:

- 1.  $[\varphi \land \psi]$  is at least as informative as  $[\varphi]$  and as  $[\psi]$
- 2.  $[\varphi \land \psi]$  is at least as attentive as  $[\varphi]$  and as  $[\psi]$
- 3. Every proposition that is at least as informative and attentive as  $[\varphi]$  and  $[\psi]$  is also at least as informative and attentive as  $[\varphi \land \psi]$ .

Floris Roelofsen



Figure 2: Three simple *might* sentences.

Fact 12 tells us that for any  $\varphi$  and  $\psi$ ,  $[\varphi \lor \psi]$  is the most homogeneous proposition that is at most as homogeneous as  $[\varphi]$  and at most as homogeneous as  $[\psi]$ . In other words,  $[\varphi \lor \psi]$  is the unique proposition with the following properties:

- 1.  $[\varphi \lor \psi]$  is at most as informative as  $[\varphi]$  and as  $[\psi]$
- 2.  $[\varphi \lor \psi]$  is at least as attentive as  $[\varphi]$  and as  $[\psi]$
- 3. Every proposition that is at most as informative and at least as attentive as  $[\varphi]$  and  $[\psi]$  is also at most as informative and at least as attentive as  $[\varphi \lor \psi]$ .

And finally, Fact 13 tells us that for any  $\varphi$ ,  $[\neg \varphi]$  is the least refined proposition whose meet with  $[\varphi]$  is  $\emptyset$ .

These algebraic characterizations give us a general understanding of the semantic behavior of the connectives in APL, and thereby provide a suitable foundation for the framework. We now turn to an illustration of how the framework may be used in natural language semantics, in particular in the analysis of attentive *might*.

# 3 Attentive might

We will first re-implement the CGR-09 analysis of attentive *might* in APL, and then compare this implementation with the original one. In Section 4 we will turn to pragmatic aspects of attentive *might*, and compare the present account with the standard modal and dynamic analyses of *might*.

### 3.1 *Might* as an attentive operator

We add an operator  $\diamond$  to our formal language, representing *might*, and define the proposition expressed by  $\diamond \varphi$  as follows.

# Definition 12 (Might).

For any  $\varphi$ ,  $[\Diamond \varphi] = [\varphi] \cup \{\omega\}$ 

Thus, in uttering  $\Diamond \varphi$ , a speaker draws attention to all the possibilities for  $\varphi$  (and to the 'trivial possibility'  $\omega$ ) without providing any information.<sup>9</sup>

<sup>&</sup>lt;sup>9</sup>Readers familiar with the standard modal analysis of *might* will get worried here: should an utterance of  $\Diamond \varphi$  not convey, at the very least, that the speaker consider  $\varphi$  possible? We take it that this is indeed conveyed, but as a conversational implicature rather than a plain semantic entailment. Section 4 shows how this implicature arises.

Floris Roelofsen



Figure 3:  $[\diamondsuit(p \lor q)] = [\diamondsuit p \lor \diamondsuit q] = [\diamondsuit p \land \diamondsuit q]$ 

To get a first impression of what this attentive treatment of *might* amounts to, let us consider three examples. First consider the proposition depicted in figure 2(a). This proposition consists of two possibilities: one possibility consisting of all worlds where p is true, and one possibility containing all possible worlds, i.e., the trivial possibility,  $\omega$ . Together, these two possibilities make up the proposition expressed by  $\Diamond p$ . Thus, in uttering  $\Diamond p$ , a speaker draws attention to the possibility that p and to the trivial possibility, without providing any information.

Next, consider the proposition depicted in figure 2(b). This is the proposition expressed by  $p \land \Diamond q$ . It consists of two possibilities: |p| and  $|p \land q|$ . Thus, in uttering  $p \land \Diamond q$ , a speaker provides the information that p holds, and draws attention to the possibility that q may hold as well.

The proposition depicted in figure 2(c) is the proposition expressed by  $\Diamond p \lor \Diamond \neg p$ . In uttering this sentence, a speaker draws attention to the possibility that p, the possibility that  $\neg p$ , and the trivial possibility, again without providing any information.

#### **3.2** How *might* interacts with the connectives

It is well-known that *might* interacts with the connectives in peculiar ways. We will consider two specific observations here, one concerning the interaction of *might* with disjunction and conjunction, and one concerning the interaction of *might* with negation. Both these observations are puzzling for the standard modal account of *might*.

**Disjunction and conjunction.** As mentioned in the introduction, Zimmermann (2000, p.258–259) observed that (6), (7), and (8) are intuitively all equivalent.

(6)	John might be in Paris or in London.	$\Diamond(p\lor q)$
(7)	John might be in Paris or he might be in London.	$\Diamond p \lor \Diamond q$
(8)	John might be in Paris and he might be in London.	$\Diamond p \land \Diamond a$

Notice that *might* behaves differently here from clear-cut epistemic modal constructions; clearly (9) is not equivalent with (10).

- (9) It is consistent with my beliefs that John is in London or it is consistent with my beliefs that he is in Paris.
- (10) It is consistent with my beliefs that John is in London and it is consistent with my beliefs that he is in Paris.

In APL, the equivalence between (6)-(8) is straightforwardly accounted for: all these sentence express exactly the same proposition, which is depicted in figure 3. Notice that, since p stands for 'John is in London' and q stands for 'John is in Paris' in this example, it is impossible for

p and q to hold at the same time. Thus, our logical space consists of three worlds in this case: one world where John is in London (10), one where John is in Paris (01), and one where John is neither in London nor in Paris (00). The proposition expressed by (6)–(8) consists of three possibilities, the possibility that John is in London, the possibility that he is in Paris, and the trivial possibility. Thus, in uttering (6), (7), or (8), a speaker draws attention to the possibility that John is in London and to the possibility that John is in Paris, without providing any information.<sup>10</sup>

**Negation.** Now let us consider how *might* interacts with negation. One striking observation is that in English, standard sentential negation cannot take wide scope over *might*. For instance, (11) can only be taken to draw attention to the possibility that John is not in London.

(11) John might not be in London.

Notice, again, that *might* behaves differently from clear-cut epistemic modal constructions here, which can very well occur in the scope of negation:

(12) It is not consistent with my beliefs that John is in London.

The fact that *might* cannot occur in the scope of negation is explained in APL by the fact that  $\neg \diamond \varphi$  is always a contradiction. On the other hand,  $\diamond \neg p$  seems to be a suitable representation of (11) in our logical language. In uttering  $\diamond \neg p$ , a speaker draws attention to the possibility that  $\neg p$ , without providing any information.

# 3.3 Comparison

Evidently, APL and CGR-09 are very much in the same spirit. Yet, there are some important differences between the two, both at the general architectural level and in terms of concrete predictions about *might*. We will first consider the general architectural differences, and then turn to some concrete predictions about *might*.

**General architectural differences.** Just as in APL, propositions are defined in CGR-09 as non-empty sets of possibilities. However, in CGR-09 propositions are not only taken to embody informative and attentive content, but also inquisitive content. As argued in the introduction, the way in which this is done is problematic because it places certain constraints on the relation between inquisitive and attentive content which do not seem warranted. Perhaps because of this fundamental problem, it has remained unclear how to give an appropriate definition of entailment in CGR-09, and as a consequence, it has been impossible to generalize the standard treatment of the connectives as expressing basic algebraic operations on propositions, e.g., conjunction and disjunction as expressing a *meet* and *join* operations, respectively. One concrete problem that can be seen as a specific result of this general deficiency is that conjunction is not idempotent in CGR-09. For instance,  $(p \lor q) \land (p \lor q)$  is not equivalent to  $(p \lor q)$ .<sup>11</sup>

APL overcomes these problems by focusing just on informative and attentive content, and leaving inquisitive content out of consideration. This way, a natural notion of entailment emerges, together with notions of homogeneity and refinement. And with these notions in place, we rather straightforwardly recover an algebraically motivated treatment of the connectives. In particular, conjunction regains idempotency under this treatment.

<sup>&</sup>lt;sup>10</sup>In Section 3.3 we will consider variants of Zimmermann's examples where p and q are not mutually exclusive. <sup>11</sup>This can be verified straightforwardly, given that disjunction is defined as union, and conjunction is defined as pointwise intersection in CGR-09.

Floris Roelofsen



Figure 4: A difference between APL and CGR-09.

Of course, this is not to say that it is impossible to have a framework in which propositions capture informative, inquisitive, and attentive content all at once. However, for the moment, if we want to capture these three types of content it seems best to adopt APL alongside the basic implementation of inquisitive semantics, which is only concerned with informative and inquisitive content. Each sentence may then be associated with two semantic values, one in APL and one in inquisitive semantics, and these two semantic values together would capture the informative, inquisitive, and attentive content of the sentence.

**Predictions about** *might*. All the examples of *might* sentences that we have seen so far express exactly the same proposition in CGR-09 as they do in APL. However, there are other sentences involving *might* that express a different proposition in CGR-09 than in APL.

Consider the following variants of Zimmermann's examples, where, unlike in the original examples, p and q are not mutually exclusive.

(13)	John might speak English or French.	$\Diamond (p \lor q)$

(14) John might speak English or he might speak French.  $\Diamond p \lor \Diamond q$ 

(15) John might speak English and he might speak French.  $\Diamond p \land \Diamond q$ 

In APL, these three sentences are still equivalent; they all express the proposition depicted in figure 4(a). In CGR-09 however, these three sentences are *not* equivalent. The first two express the proposition depicted in figure 4(a), while the third, involving conjunction, expresses the proposition depicted in figure 4(b).

Ciardelli *et al.* (2009) argue that this prediction is in fact desirable, based on a scenario suggested by Anna Szabolcsi. The scenario is one in which someone is looking for an English-French translator, i.e., someone who speaks *both* English and French. In that context, (15) would be perceived as a useful recommendation, while (13) and (14) would not. Now, in CGR-09,  $\Diamond p \land \Diamond q$ , unlike  $\Diamond (p \lor q)$  and  $\Diamond p \lor \Diamond q$ , draws attention to the possibility that  $p \land q$ , that is, the possibility that John speaks both English and French. This, then, could explain the observation that (15) is perceived as a useful recommendation in the translator-scenario, unlike (13) and (14).

In APL, this explanation is no longer available. However, there are reasons to be skeptical about the prediction made by CGR-09. For instance,  $\Diamond p \land \Diamond q$  is predicted to express exactly the same proposition as  $\Diamond (p \lor q \lor (p \land q))$ . However, as noted by Luis Alonso-Ovalle (p.c.), in English there are clear differences between these sentences:

(16) ✓ John might speak English and he might speak French, but of course he doesn't speak both.

(17) #John might speak English or French, or both, but of course he doesn't speak both.

This contrast is consistent with APL, where  $\Diamond (p \lor q \lor (p \land q))$  draws attention to the possibility that  $p \land q$ , but  $\Diamond p \land \Diamond q$  doesn't. However, in CGR-09 the contrast cannot be explained, since both  $\Diamond p \land \Diamond q$  and  $\Diamond (p \lor q \lor (p \land q))$  draw attention to the possibility that  $p \land q$ .

Thus, in Szabolsci's scenario CGR-09 seems to make better predictions than APL, but for Alonso-Ovalle's examples the predictions of APL seem to be more appropriate. Below we will suggest two ways to ameliorate the predictions of APL in Szabolsci's scenario.

**Concord.** One option would be to assume that *might*, in English, makes no direct semantic contribution. Instead it signals that there is some operator—call it  $\Sigma$ —higher up in the syntactic tree, which is interpreted as  $\diamond$ , and stands in a syntactic agreement relation with *might*. Crucially, we may assume that  $\Sigma$  can agree with multiple occurrences of *might* in its scope. Similar proposals have been made in the literature for negation and modals, under the heading of *negative* and *modal concord*, respectively (see, for instance Zeijlstra, 2004, 2007, 2008). Under these assumption about *might*, sentences like (18) are structurally ambiguous: depending on their underlying syntactic structure, they could be translated into our logical language either as (18-a) or as (18-b).

(18) John might speak English and he might speak French.

a.  $\Diamond p \land \Diamond q$ b.  $\Diamond (p \land q)$ 

Of course, sentences like (19) would also be structurally ambiguous, with (19-a) and (19-b) as possible translations depending on the underlying syntactic structure.

(19) John might speak English or he might speak French.

a.  $\Diamond p \lor \Diamond q$ b.  $\Diamond (p \lor q)$ 

However, we have seen that (20-a) and (20-b) are semantically equivalent. So in the case of (20), the presumed structural ambiguity does not give rise to a semantic ambiguity.

The analysis sketched here would allow us to explain Szabolsci's observation: if (18) is interpreted as in (18-b), it draws attention to the possibility that John speaks both English and French, which makes it a useful recommendation in Szabolsci's scenario. This does not hold for (19), or for the variant where disjunction takes low scope under *might*, because these sentences do not draw attention to the possibility that John speaks both English and French, no matter what their underlying syntactic structure is.

At the same time, the analysis is compatible with Alonso-Ovalle's observation, since (18), on one of its syntactic analyses, does *not* draw attention to the possibility that John speaks both English and French, and therefore differs semantically in the relevant respect from *John might speak English or French or both*.

**Subordination.** Another option would be to enrich the semantic apparatus, rather than the syntactic assumptions. In particular, we could assume that *might* sentences, besides drawing attention to certain possibilities, also make these possibilities available as hypothetical contexts relative to which subsequent sentences may be evaluated. This phenomena is known as *modal subordination* (see, for instance Roberts, 1989; Kaufmann, 2000; Brasoveanu, 2007). A

detailed implementation is beyond the scope of this paper, but it can be expected that such an enrichment of the semantic framework would naturally lead to an account of Szabolsci's and Alonso-Ovalle's observations.

# 4 Pragmatics

Standard Gricean pragmatics assumes a classical, truth-conditional semantics, where the meaning of a sentence is identified with its informative content. In APL, semantic meaning is not identified with informative content; rather, it encompasses both informative and attentive content. This semantic refinement also changes our perspective on pragmatics.<sup>12</sup> This new perspective on pragmatics, and the particular consequences that it has for the interpretation of *might*, were already explored in Ciardelli *et al.* (2009, 2010). We will adapt the ideas established there to the present setting. We will restrict our attention to pragmatic considerations that are directly relevant for *might*. A more comprehensive attention-sensitive pragmatics has been developed in recent work by Westera (2013) and related ideas are also being pursued in ongoing work by Van Rooij (2013).

# 4.1 Sincerity and transparency

Consider a conversation in which the participants' main purpose is to exchange information in order to resolve a given issue as effectively as possible. In such a cooperative effort, each participant must be *sincere*. In the present setting, this sincerity requirement has an informative and an attentive component. On the one hand, a speaker who utters a sentence  $\varphi$  must take herself to know that the actual world lies in  $info(\varphi)$ . We will call this *informative sincerity*. On the other hand, a speaker who draws attention to a certain possibility must consider this possibility a 'live possibility:' it must be consistent with her information state. This we will call *attentive sincerity*.

Participants must also be *transparent*. That is, if one participant draws attention to a certain possibility, and this possibility is inconsistent with the information state of another participant, then this other participant must publicly announce this inconsistency, so that other participants will refrain from considering the possibility in question. Notice that the sincerity requirement is *speaker* oriented, while the transparency requirement is *hearer* oriented.

Besides these qualitative sincerity and transparency requirements, there are also certain *quantitative preferences*. In particular, among all the sentences that could be sincerely uttered and that would be relevant for resolving the given issue under discussion, there is a general quantitative preference for *more informative* sentences—the more relevant information one provides, the more likely it is that the given issue will be resolved.

Without going into the more subtle details, let us lay out the basic repercussions of a pragmatic theory along these lines for the interpretation of *might*.

# 4.2 Quality implicatures

There are two empirical observations about *might* that we have not discussed at all so far, even though each of them has given rise to one of the two most prominent semantic theories of *might*. Both observations can be illustrated by means of the following minimal example:

 $<sup>^{12}</sup>$ This is analogous to the way in which inquisitive semantics, by giving an inquisitive twist to the notion of semantic meaning, gives rise to a richer perspective on pragmatics (see Groenendijk and Roelofsen, 2009).

### (20) John might be in London.

The first observation, perhaps the most basic one, is that if someone utters (20) we typically conclude that she considers it *possible* that John is in London. This observation has given rise to the commonplace analysis of *might* as an epistemic modal operator.

The second observation is that if someone hears (20) and already knows that John is not in London, she will typically object, pointing out that (20) is inconsistent with her information state. In this sense, even though *might* sentences do not provide any information about the state of the world, they can be 'inconsistent' with a hearer's information state. One prominent account of this observation is that of Veltman (1996). Veltman's update semantics specifies for any given information state  $\sigma$  and any given sentence  $\varphi$ , what the information state  $\sigma[\varphi]$  is that would result from updating  $\sigma$  with  $\varphi$ . The update effect of  $\Diamond \varphi$  is defined as follows:

$$\sigma[\Diamond \varphi] = \begin{cases} \emptyset & \text{if } \varphi \text{ is inconsistent with } \sigma\\ \sigma & \text{otherwise} \end{cases}$$

The idea is that, if  $\varphi$  is inconsistent with a hearer's information state, then updating with  $\Diamond \varphi$  leads to the absurd state. To avoid this, the hearer must make a public announcement signaling the inconsistency of  $\varphi$  with her information state. As a result, whoever uttered  $\Diamond \varphi$  in the first place may also come to discard the possibility that  $\varphi$  holds.

Our semantics does not directly explain these observations. However, we believe that this is rightly so. In our view, both observations should be explained pragmatically. And they can be. It follows from the attentive sincerity requirement that if a cooperative speaker utters a sentence  $\varphi$  and  $\alpha$  is a possibility in [ $\varphi$ ], then  $\alpha$  must be consistent with the speaker's information state. In particular, a cooperative speaker who utters (20) must consider it possible that John is in London.<sup>13</sup>

On the other hand, it follows from the transparency requirement that if a hearer is confronted with a sentence  $\varphi$ , and one of the possibilities for  $\varphi$  is inconsistent with her information state,

- (i) a. John might be in London or in Paris.b. John is in London or in Paris.
- b. John is in London of in Faris.
- (ii) a. John is somewhere in Europe.b. Where is John?

The sentences in (i) license the inference that the speaker considers it possible that John is in London and that she considers it possible that John is in Paris. The sentences in (ii) however, do not license this inference: a

she considers it possible that John is in Paris. The sentences in (ii) however, do not license this inference: a cooperative speaker who utters these sentences may very well know that John is not in London or in Paris. This is surprising under the assumption that indefinites and constituent questions draw attention to possibilities, just like disjunction and *might* sentences, and that the attentive sincerity requirement applies to each of these possibilities.

There are at least two possible ways to explain the contrast between (i) and (ii). First, the indefinite in (ii-a) and the question in (ii-b) are quantificational operators, and the domain that they quantify over is generally understood to be implicitly restricted. Thus, we cannot tell from the surface form of these sentences whether or not the intended domain of quantification contains Paris and/or London. Hence, the relevant inference does not arise. Notice that the constructions in (i) do not involve quantification. Thus, in these cases the inference cannot be blocked by uncertainty regarding the domain of quantification.

Another factor that plausibly plays a role is *efficiency*. Consider a speaker who knows that John must be somewhere in Europe, but not in Paris, Barcelona, Rome, Prague, Vienna, or Berlin. Such a speaker could choose to ask the question in (ii-b) without explicitly stating that she already knows that John is not in any of the mentioned cities. Strictly speaking, this move is not fully cooperative. However, this is outweighed by the fact that the fully cooperative alternative move is highly inefficient. This is different for, say, (i-b). In this case, the more cooperative alternative, which is just to state that John is in London, would also be more efficient.

 $<sup>^{13}</sup>$ It must be noted that the attentive sincerity requirement is sometimes 'neutralized' by other pragmatic factors. To see this, consider the sentences in (i) and (ii):

then she must signal this inconsistency, in order to prevent other participants from considering the possibility in question a 'live option.'

Thus, both observations are accounted for. And this pragmatic account, unlike the mentioned semantic analyses, extends straightforwardly to more involved cases. Consider for instance:

(21) John might be in London or in Paris.

This sentence is problematic for both semantic accounts just mentioned. The epistemic modality account predicts that the speaker considers it possible that John is in London or in Paris. But note that this is compatible with the speaker knowing perfectly well that John is not in London. What (21) implies is something stronger, namely that the speaker considers it possible that John is in London *and* that she considers it possible that John is in Paris. This follows straightforwardly on our pragmatic account.

Now consider a hearer who is confronted with (21) and who knows that John is possibly in Paris, but certainly not in London. We expect this hearer to object to (21). But Veltman's update semantics does not predict this: it predicts that an update with (21) has no effect on the hearer's information state. Our pragmatic account on the other hand, does urge the hearer to object.

The only task of our semantics is to specify which propositions are expressed by which sentences. The pragmatics, then, specifies when a speaker is licensed to utter a certain sentence, and how a hearer is supposed to react to a given utterance. Together, these two components account for the basic features of *might* that the most prominent semantic theories take as their point of departure. Shifting some of the weight from semantics to pragmatics evades problems with more involved cases, like (21), in a straightforward way. But, of course, the necessary pragmatic principles can only be stated if the underlying semantics captures more than just informative content.

# 4.3 Quantity implicatures

If someone says that John might be in London, we typically do not only conclude that she considers it possible that John is in London, but also that she considers it possible that he is *not* in London. In short, we infer that the speaker is *ignorant* as to whether John is in London or not.

This implicature is straightforwardly derived. We have already seen how to establish the inference that the speaker considers it *possible* that John is in London. Moreover, it follows from the quantitative preference for more informative sentences that whenever a cooperative speaker S utters a sentence  $\varphi$  and  $\alpha$  is a possibility in  $[\varphi]$  such that  $\alpha \subset \inf(\varphi)$ , we can conclude that S does not have sufficient information to sincerely utter a sentence  $\varphi'$  expressing the proposition  $\{\alpha\}$ . After all, assuming that  $\varphi$  is relevant w.r.t. the given question under discussion,  $\varphi'$  would also be relevant w.r.t. the given question under discussion, under any sensible notion of relevance. Thus, the only possible reason why S did not directly utter  $\varphi'$  instead of the less informative  $\varphi$  is that her information state does not support the informative content of  $\varphi'$ . In other words, she is not certain whether the actual world is contained in  $\alpha$ .

#### 4.4 Epistemic re-interpretation

In certain embedded environments,  $\Diamond p$  really seems to be interpreted as saying that p is consistent with some contextually given body of information (usually, but not necessarily, the

information state of the speaker). For instance, (22) univocally conveys that the speaker believes that John will not go to London.

(22) It is not true that John might go to London.

If the sentence is analyzed as  $\neg \diamondsuit \varphi$ , then it is predicted to be a contradiction in APL, which is evidently not the right prediction.

One may be tempted to conclude that this simply shows that *might* is ambiguous, permitting both an 'epistemic use' and an 'attentive use,' and possibly other usages as well. However, it may be worth trying to avoid such a conclusion, at least in its strongest form. For, if *might* were simply ambiguous between an attentive use and an epistemic use, then we would lose our explanation for the fact that *might* obligatorily takes wide scope over standard negation, unlike modal constructions like 'it is consistent with my beliefs that.' Recall the relevant example:

(23) John might not go to London.

In Section 3.2, we offered an explanation for the fact that negation cannot take wide scope in (23), which is that  $\neg \diamond p$  always amounts to a contradiction. But of course this explanation only goes through if the semantic contribution of  $\diamond p$  indeed univocally lies in its potential to draw attention to the possibility that p. If  $\diamond p$  were ambiguous, and could also be interpreted semantically as saying that p is consistent with some contextually determined body of information, then there would be no reason anymore why negation should obligatorily take narrow scope. After all, we saw that negation is perfectly happy with wide scope in sentences like (24):

(24) It is not consistent with my beliefs that John will go to London.

Thus, rather than assuming plain ambiguity, we would like to offer a more nuanced account of the epistemic interpretation of *might* in (22). Notice that in this particular case there is a specific *reason* not to adopt the standard interpretation of  $\Diamond p$ . We hypothesize that this triggers *re-interpretation* of  $\Diamond p$  in terms of the implicatures that it typically triggers when not embedded.<sup>14</sup>

More specifically, we hypothesize that (22) is interpreted as a denial of one or more implicatures of the embedded clause. It is in fact a common use of 'it is not true that' constructions to deny pragmatic inferences or presuppositions of their complement clause. For example, in (25) the implicature of the embedded clause is denied, and in (26) the presupposition of the embedded clause is denied:

- (25) It is not true that John has four children. He has five.
- (26) It is not true that the king of France is bald. There is no king of France.

Notice that (22) is not necessarily interpreted as denying that it is *possible* that John will go to London. It may also be interpreted as denying the stronger implicature that it is *unknown* whether John will go to London or not. For, someone who utters (22) may continue as in (27), but also as in (28) (where SMALLCAPS indicate contrastive stress).<sup>15</sup>

 $<sup>^{14}</sup>$ The proposal made here is in line with recent observations by Levinson (2000) and Chierchia, Fox, and Spector (2011), among others, that the semantic contribution of certain expressions is sometimes strengthened 'locally', i.e., before it enters the semantic composition process. Construing this process as 're-interpretation' is especially in line with Geurts' (2009) take on such phenomena.

 $<sup>^{15}</sup>$ In (28) and (30), it is strongly preferred, perhaps even necessary, to not only place contrastive stress on *will* and *both*, but also on *might* and *or*. This observation does not seem to affect our argument however. See (Fox and Spector, 2009) for relevant discussion.

Floris Roelofsen

- (27) It is not true that John might go to London. He will go to PARIS.
- (28) It is not true that John might go to London. He WILL go to London.

Notice that a similar pattern arises with disjunction:

- (29) It is not true that John speaks English or French. He speaks NEITHER.
- (30) It is not true that John speaks English or French. He speaks BOTH.

These observations support the idea that 'it is not true that' constructions can be interpreted as denying pragmatic inferences that the embedded clause gives rise to, and thus lend support to a re-interpretation analysis of examples like (22).

One may ask, of course, why this same re-interpretation strategy could not be applied in (23). The answer would be that re-interpretation only occurs if it is triggered. In (23), negation can take narrow scope, and the interpretation of  $\Diamond \neg p$  is unproblematic. Thus, there is no need for re-interpretation. In (22) however, negation is forced to take wide scope, and  $\neg \Diamond p$  is, at face value, a contradiction. This is what triggers re-interpretation in this case.

In Ciardelli *et al.* (2010) it is argued that this explanation also extends to the interpretation of *might* clauses embedded under a question operator or in the antecedent of a conditional. We hypothesize, therefore, that non-attentive readings of *might* are generally the result of reinterpretation. More work is needed, of course, to solidify this claim. But we think this is a direction worth pursuing.<sup>16</sup>

# 5 Final remarks

This paper has given concrete shape to the idea that might sentences are primarily used to draw attention to certain possibilities, an idea that goes back (at least) to Groenendijk, Stokhof, and Veltman (1996). We have seen that this can be done while preserving our conception of possible worlds as total objects, and still explicating information growth in terms of elimination of possible worlds, something that Groenendijk, Stokhof, and Veltman considered unachievable. Our approach heavily builds on the work of Ciardelli, Groenendijk, and Roelofsen (2009). In fact, the essence of the analysis of *might* developed there is fully preserved. However, it is implemented in a more minimalistic semantic framework, in order to overcome some of the problems, both foundational and empirical, that the original implementation faces.

It is perhaps worth emphasizing that, even though this paper focuses on characterizing the possibilities that *might* sentences draw attention to, we certainly do not think that this is all there is to the meaning of *might*. Drawing attention to possibilities may have several side-effects. We discussed how ignorance implicatures typically enter the picture through (possibly conventionalized) pragmatic reasoning. Another potential side-effect of drawing attention to a certain possibility, also briefly mentioned at the end of Section 3, is the introduction of a *hypothetical context*, an idea that is familiar from the literature on modal subordination (Roberts, 1989; Kaufmann, 2000; Brasoveanu, 2007, among others).

Finally, we would like to emphasize that the framework developed in this paper potentially has many more applications that the analysis of attentive *might*. One other domain where attentive content seems to play a crucial role is that of *evidentials*. For instance, certain types of evidentials are described in the literature as 'presenting' a certain proposition, without

 $<sup>^{16}</sup>$ A weaker hypothesis that may be worth considering is that the attentive use of *might* is historically primary, and that non-attentive usages are derivative, though (partly) grammaticized (in the general spirit of, e.g., Levinson, 2000).

establishing whether that proposition holds or not (see, for instance, Faller, 2002; Murray, 2010). These are precisely the type of empirical findings that the framework developed in this paper could help to elucidate.

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