A Note on Truthlikeness and Irrelevance

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Abstract

Parameterized verisimilar or truthlike orders are concerned with ordering theories relative to a third theory. We argue that not only should irrelevant consequences of the theories to be ordered be disallowed, but so should irrelevant consequences of the theory relative to which the order is defined.

One of the many topics Johan van Benthem has contributed to is that of truthlikeness, or verisimilitude. This notion was introduced by Karl Popper (1963), as a necessary ingredient in his philosophy that science makes progress by discarding one theory in favour of another which is closer to the truth. On Popper's definition, Theory A is closer to the truth than Theory B if and only if it has more true consequences and fewer false consequences. However, David Miller (1974) and Pavel Tichý (1974) showed that Popper's ordering on theories was flawed: on his definition false theories are not comparable by verisimilitude at all. This creative error gave rise to a whole literature on what Miller formulated as *the problem of verisimilitude*: 'What can there be about one false theory that makes it closer to the truth than is another?'

In this Note we observe that whatever the answer may be, irrelevant facts should not be part of it.

To begin with, some background. An initial phase of the verisimilitude investigation culminated in three books: (Oddie, 1986), (Niiniluoto, 1987) and the collection (Kuipers, 1987b). Later developments are surveyed in (Niiniluoto, 1998) and (Zwart, 1998). Following Zwart, we may distinguish between *content* proposals and *likeness* proposals. Content proposals place the emphasis on the logical strength of theories. Likeness proposals place the emphasis on the truth or falsity of the atoms of the language, and accordingly, on every model of the theory. Our comments feature in the context of likeness proposals.

The problem of verisimilitude, as conceived by Popper, deals with ordering theories relative to a theory which pronounces upon the truth or falsity of all facts. This *complete truth assumption* has not been shared by all subsequent authors on verisimilitude. Dropping the complete truth assumption yields a parameterized 'theorylike' order where two theories A and B are ordered relative to a given third theory C, which could be construed as a belief set of some kind. Some examples of such parameterized orders in the literature are: Kuipers' (1987a; 1992) naive and refined structuralism, Niiniluoto's (1987) truthlikeness for singular sentences, van Benthem's (1987) investigation into the link between verisimilar orders and conditional assertions, and our own generalization (Britz & Brink, 1995) of an earlier proposal by Brink and Heidema (Brink & Heidema, 1987). In what follows, we deal with such a parameterized order.

The question of how the notion of relevance interacts with that of verisimilitude has been raised before, in (Schurz & Weingartner, 1987), but then in the context of a content-based verismilar order, and the complete truth assumption. Schurz and Weingartner argued that irrelevant consequences of the theories to be ordered should be disallowed. Our task is to make somewhat the same observation for likeness proposals, and without the complete truth assumption. Simply put: we argue that not only should irrelevant consequences of the theories to be ordered be disallowed, but so should irrelevant consequences of the theory (or belief set) relative to which the order is defined. This is not the case in all proposals, as we will demonstrate for Kuipers' refined structuralist approach (Kuipers, 1992; Kuipers, 1997).

Kuipers' 'naive' structuralist definition of truthlikeness is formulated in terms of set-theoretic structures. It applies readily to a model-theoretic treatment of truthlikeness of propositional structures (which is what we wish to use for our illustrations). Namely, let L denote the language of classical propositional logic. A structure is identified with a propositional constituent, i.e. a conjunction of all elementary propositions in the language, either negated or unnegated. Let Γ denote an arbitrary, fixed set of statements from L, relative to which the order will be defined. A model of Γ is a propositional valuation which satisfies every element of Γ, with $Mod\Gamma$ denoting the set of models of Γ. The symmetric difference between two valuations u and w is the set of elementary propositions on which they differ, written $u -_s w$. The models of Γ represent *empirical possibilities*, and the non-models empirical impossibilities. A *theory* Φ is considered to be a formula set, together with the claim $Mod\Phi = Mod\Gamma$. Thus the theory Φ is true iff it has exactly the same empirical possibilities as Γ. In approximating Γ, Φ can make two kinds of mistakes: it can exclude some empirical possibilities, and it can include some empirical impossibilities. These two sets are given by $Mod\Gamma - Mod\Phi$ and $Mod\Phi - Mod\Gamma$ respectively. A theory approximation Ψ is said to be closer to the theory Γ than a theory Φ if it makes fewer mistakes of each kind.

The 'refined' structuralist approach to truthlikeness resembles the naive definition, but now the order is based on an underlying ternary relation of *structurelikeness*, which indicates which of two structures is more similar to a third. In the propositional context, the relation determines which of two valuations is closer to a third valuation. Kuipers (1992) proposes that the relation of structurelikeness be defined in terms of the symmetric difference relation. Thus, for any valuations u, v and w, w is at least as similar to v as u is, iff $w -_s v \subseteq u -_s v$. A second component in the refined structuralist definition is that of relatedness or comparability of structures, but since all valuations are comparable by the symmetric difference relation, this component trivialises. The refined structuralist order of truthlikeness then states that Ψ is at least as close to Γ as Φ is, iff every approximation of an instantial match made by Φ can be improved upon by an approximate instantial match made by Ψ, and every explanatory mistake made by Ψ is an improvement of some explanatory mistake made by Φ.

Definition 1 *Given any formula sets Γ, Φ and Ψ, Ψ is at least as close to Γ as Φ is, written $\Phi \sqsubseteq_\Gamma \Psi$, iff*

(Ri) $(\forall u \in Mod\Phi)\,(\forall z \in Mod\Gamma)\,(\exists w \in Mod\Psi)\,[w -_s z \subseteq u -_s z]$, *and*

(Rii) $(\forall w \in Mod\Psi - (Mod\Phi \cup Mod\Gamma))\,(\exists u \in Mod\Phi - Mod\Gamma)$
$(\exists z \in Mod\Gamma)\,[w -_s z \subseteq u -_s z]$.

We now give two examples of how irrelevance can lead to counterintuitive results. Both examples are phrased in terms of a belief set formulated in a universe consisting of three facts, p, q and r. The structurelikeness relation is given by the usual Boolean ordering on 3-tuples of propositional valuations.

As a first example, consider the belief set $\Gamma = \{p \wedge q\}$. The set of models of Γ is $Mod\Gamma = \{110, 111\}$. Given a valuation v, the value assigned by v to r is irrelevant to whether v is a model of Γ, and should therefore also be irrelevant to how close v is to being a model of Γ. This is, however, not the case. For suppose we want to order the statements $\phi = p \wedge \neg q \wedge r$ and $\psi = \neg p \wedge \neg q \wedge \neg r$ relative to Γ. Intuitively, ϕ should be closer to Γ than ψ is, since ϕ agrees with Γ on p, whereas ψ disagrees on every atomic claim made by Γ. But, since $101 -_s 110 \not\subseteq 000 -_s 110$, (Ri) pronounces that $\psi \not\sqsubseteq_\Gamma \phi$. This is because (Ri) looks at each constituent of Γ separately when seeking to improve the instantial match 000. And while every atom in the language is relevant when considering a single constituent of Γ, some parts of the constituent (in this case, the truth value of r) may actually be irrelevant. This irrelevance of r is only expressed by Γ as a whole.

As a second, slightly more subtle example, consider the belief set $\Gamma' = \{p \vee q, \neg r\}$. The models of Γ' is the set $Mod\Gamma' = \{110, 100, 010\}$. Suppose we want to order the wffs $\phi = p \wedge \neg q \wedge r$ and $\zeta = p \wedge q \wedge r$. Since $111 -_s 100 \not\subseteq 101 -_s 100$, (Ri) pronounces that $\zeta \not\sqsubseteq_\Gamma \phi$. This is again counter-intuitive: q does not appear negated in Γ, and an increase in the truth value of q should therefore make 111 at least as close to being a model of Γ than 101. Similarly, (Rii) states that every explanatory mistake w made by Ψ is not as bad as some explanatory mistake u made by Φ. The criterion used to determine this, is the existence of a model z of Γ which is closer to w than to u. Kuipers (1997) later proposed that the existence criterion be strengthened to z to be a model of $\Gamma - \Phi$, but this also does not address the problem of relevance. The relatedness relation also does not suffice, since it, too, is defied on individual constituents.

Kuipers' first order criterion, which considers constituents separately, does not capture the notion of relevance to Γ as a whole. For this, one needs to define an order $\leq_\Gamma$ on valuations, relative to Γ. In (Britz & Brink, 1995) we did so, aiming (amongst other things) to take seriously the issue of relevance to Γ. Without going here into the technical details of how the order on valuations is defined, we observe only that the actual parameterized verisimilar ordering is then introduced in the 'power relation' style of Brink and Heidema (1987). As follows:

Definition 2 *Given any formula sets Φ, Ψ and Γ, Ψ is at least as close to Γ as Φ is, written $\Phi \leq_\Gamma^+ \Psi$, iff*

(Ti) $(\forall v \in Mod_{\mathrm{PC}}\, \Phi)\,(\exists w \in Mod_{\mathrm{PC}}\, \Psi)\,[v \leq_\Gamma w]$, *and*

(Tii) $(\forall w \in Mod_{\mathrm{PC}}\, \Psi)\,(\exists v \in Mod_{\mathrm{PC}}\, \Phi)\,[v \leq_\Gamma w]$.

This ordering deals effectively with the problem of irrelevance – at least in the sense of not being subject to the kind of examples of counterintuitive results given above. The reader who wishes to check that this is the case could consult the exposition in (Britz & Brink, 1995).

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