Some Logic in Communication

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Abstract

Sentences containing pronouns or indefinite noun phrases can be said to express open propositions, propositions which display gaps to be filled. This paper addresses the question what kind of information of a speaker can be said to support the utterance of such propositions. We consider a set of cases exemplifying proper and improper uses of these terms, and we argue that neither the classical ('objective'), nor the discourse-oriented or dynamic (hearer's) perspective on (variable) meaning give us a straightforward answer to this question.

We then motivate and formulate a speaker's perspective on meaning, and relate it to the classical and the dynamic one. We propose three key notions, that of the content of a sentence, that of the support for the utterance of a sentence, and that of the update which an utterance may bring about. The three notions are each defined in a compositional fashion and brought together within a single coherent framework.

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1 General Background

Since the work of Gottlob Frege and Bertrand Russell it is relatively generally acknowledged that there are two types of noun phrases in natural language. Referential expressions like proper names and definite descriptions can be assumed to refer to individuals; quantifying expressions, noun phrases headed by determiners like *every*, *no*, *most*, etc., do not refer, but predicate something of the set associated with the verb phrase with which they are combined. However, there are two types of terms, indefinite descriptions and pronouns, which do not so easily fit in this scheme of classification. Syntactically, they behave on a par with proper names and definite descriptions; semantically, however, they do not, always and obviously, refer.

Indefinites and pronouns have been dealt with extensively in systems of discourse interpretation (Hans Kamp, Irene Heim) and dynamic semantics (Jeroen Groenendijk and Martin Stokhof), where they are assigned a special discourse role. These systems model the epistemic content or cognitive value of sentences in a discourse and interpret indefinites and pronouns as so-called 'discourse referents'. These discourse referents behave like free variables and they can be interpreted referentially and they can be quantified away.

In the discourse-oriented dynamic analysis natural language is approached from a hearer's perspective. In these systems the updates are modeled which utterances or assertions may bring about in the information states of interpreters. Thus, these systems leave two questions unanswered. Firstly, what kind of information may a *speaker* adduce or appeal to in order to support his or her utterances? Secondly, what is the *meaning* of a free variable? It may be obvious that the first question asks for a further qualification of, e.g., Grice's maxims of quality or Hamblin's commitment slates. The second question—posed by Fred Landman with a capital questionmark—must, apparently, be answered in order to find out what the meaning of indefinites is upon the discourse or dynamic analysis.

2 The Meanings of Variable Sentences

The question what is the meaning of sentences containing indefinite elements, or variables, has received considerable attention from philosophers and linguists. An obvious answer to this question is: "that depends on what the variable refer to," and this answer has been generalized, in the spirit of Tarski and Lewis, by Theo Janssen. The meaning of a sentence with free variables is a function, which defines, for any possible valuation of the variables, which truth value the sentence has under that valuation, or which proposition it then expresses.

This outlook upon the meaning of sentences with variables seems to be appropriate for (formal) semantic systems, but it raises a non-trivial question when it comes to pragmatic issues. Think of Grice's quality maxims, which, roughly, say that a speaker should have information supporting the propositions she utters. Like we said, sentences with indefinite elements do not express propositions, or, rather, they can be taken to express a whole bunch of them, depending on what value is assigned to their open places. Should a speaker support all of these propositions, or at least one of them? Clearly these would be unnatural and inadequate requirements, respectively.

At this point it seems plausible to require a speaker to have information supporting the proposition which is expressed relative to the valuation of the open places which the speaker has in mind. This, however, is also not unproblematic. Pronouns may be coreferential with indefinite expressions, and these do not seem to have a unique semantic referent. And, again, it would be unnatural or inadequate to require a speaker to support the open proposition relative to all valuations of the pronoun which agree with a possible valuation of the antecedent, or with at least one of them. Nevertheless, we think the suggestion inolving speaker's reference comes close to what seems to be required.

3 Linguistic Data

We proceed with an empirical survey of the semantic / pragmatic data involving the use of indefinite noun phrases and pronouns. Consider the following sample of (multi-speaker) discourse:

(1) *Nel:* Yesterday a member of parliament came to see the queen.

(2) *Len:* (I heard.) He was dead drunk!

In this piece of dialogue, Len's reply is OK if he has good motivation for assuming that the same gossip was reported to him which Nel wants to report, and that he and Nel refer to the same member of parliament. Notice that neither Nel nor Len must be assumed to know which drunk member of parliament it was who paid a visit to the queen, if there was one anyway. Things are alright if it is reasonable for Len to assume that Nel is about to report smalltalk about the same individual he has heard of. (And he may be wrong, also if a drunk member of parliament did visit the queen. Nel might correct him by saying *No he was not. Not the one I meant.* He *was stoned.*)

The next example can be assumed to be uttered by two agents who perceive the same visual scene in a park:

(3) *Sim:* A man is sleeping on a park bench.

(4) *Mos:* It is not a man, it is a woman, and she is not asleep, she is just sunbathing. Besides it is not a park bench.

This exchange is fine only if *Mos* has evidence that *Sim* used the indefinite *a man* with reference to an entity which *Mos* thinks to be a woman, who is sunbathing on something which is not a parkbench. Notice that Mos's reply is motivated if he has sufficient reason to believe these things of an individual he thinks Sim refers to. Not one of Sim's original attributions to this person appear to be relevant.

Finally consider the following example:

(5) *Liz:* Yesterday, a man ran into my office who inquired after the secretary's office.

(6) *Dib:* Was he wearing pink pumps?

(7) *Liz:* I don't know. If it was Wilburt he was, if it was Norbert, he was not.

Liz's reply is odd if she is aware that only Wilburt and Norbert came to her to inquire after the secretary's office, and if she knows the first did and the second did not wear pink pumps. If she knows whom she is talking about, she could have given a more definite reply. Nevertheless, she could motivate her reply saying that she had started a report about a man who took her keys from her office yesterday, which must have been either Wilburt or Norbert, and that she did not know which of the two it was.

4 Outline of the Proposal

The above examples seem to motivate the following generalizations regarding the exchange of open propositions:

- indefinite terms can be used with referential intentions
- anaphoric or coreferential pronouns relate to the individuals which their antecedents are assumed to refer to

These requirements involve vague notions, the specification of which is complicated by two facts. Firstly, it may be unclear, and semantically underspecified, which individual, if any, it is that someone intends to refer to. Secondly, each interlocutor has his own (private) representation of the individuals under discussion. The full analysis is therefore based on two additional assumptions:

1. the information states of agents are structured around belief objects; the presence of a belief object in an information state models the assumption that it represents a unique individual
2. some of these belief objects actually are related to unique individuals by perceptual and intentional links or chains

An indefinite term can then be said to be properly used by a speaker only if she uses it with a specific belief object in mind, which represents a unique individual for her. A subsequent pronoun, uttered by her or by a respondent, can be coreferential then if the utterer has an object in mind which he knows as the intended referent of the antecedent term.

5 Formal Framework

Finally we connect things up in formal models for information exchange. We motivate and define basic formal notions like the following:

- information aggregates (which include information states, and linguistic meanings)
- subjects of aggregates (which include belief objects and discourse referents)
- the support of information and the product of information of sets of aggregates

Our aggregates are Heim-style *files* (interpreted *discourse representations*): they are sets of assignments of values to (finite) sets of variables. The notions of support and information product are defined relative to functions linking the variables of different aggregates. The definitions can be found in an appendix to this abstract.

We also give a compositional characterization of (i) the contents, (ii) the update values, and (iii) the support requirements of a designed (first order) language. We show the three to be formally and empirically well-behaved in the following sense: relative to the mentioned links, the update of an information state τ with a sentence S is seen to equal the information product of τ with the meaning of S; and also relative to these links, an utterance of S is seen to be licensed by an information state σ iff σ supports S's meaning. Thus we provide for a general framework in which we can characterize safe exchange of first order information by means of indefinite and pronominal expressions.

6 Conclusion

In this paper we have studied the use of indefinites and pronouns from a speaker's perspective. We have argued that both are used with referential intentions, which are mediated by the speaker's representation of individuals in the domain of discourse. These referential intentions also motivate the discourse referents which these terms give rise to in systems of discourse representation and dynamic semantics. A discourse referent serves to represent for the hearer the individual which the speaker had in mind when using such a term. The hearer may also take this individual in mind, for instance, when he communicates what he has learned from the speaker to a third party.

The issue of the support of indefinites came up in response to a question concerning the meaning of indefinites and pronouns. Indefinites and pronouns are like free variables, and the meanings of sentences with indefinite expressions can in general be equated with functions from the possible values of these indefinites, to the propositions which are expressed under these valuations. This notion of meaning, as well as that of the update value and that of the support requirements of a sentence, have all been given a compositional definition within a formal logical framework of information.

Our perspective upon the use of indefinites also applies to other uses of referential terms. People may exchange information using definite and indefinite noun phrases, names and pronouns to refer to individuals they do or do not know themselves. One may say that all these terms behave like free variables, which (ought to) relate to the belief-objects of a speaker. The various type of terms only seem to differ in their presuppositions and their behavior in the scope of certain constellations. For instance, indefinites may be bound by quantifying adverbs like *always* and *usually*, or by a negation. Proper names and pronouns can not be bound thus. Furthermore, indefinites and pronouns can be bound locally, by adverbs and antecedent noun phrases, whereas proper names ought to relate to individuals familiar to the interlocutors.

7 Appendix

We formulate basic notions of meaning, support and update for a language of first order predicate logic extended with anaphoric pronouns $(\mathtt{p}_1, \mathtt{p}_2, \ldots)$. The indices on the pronouns indicate how many terms back in the discourse their antecedent has to be found. We start with the definition of a Tarskian notion of satisfaction, for which we need to refer to the length and the scope of our formulas:

Definition 1 (*PLA*: Length $n(\phi)$ and Scope $s(\phi)$ of a Formula ϕ)

- $n(Rt_1 \ldots t_m) = 0 \quad n(\neg\phi) = 0$
  $n(\exists x\phi) = n(\phi) + 1 \quad n(\phi \wedge \psi) = n(\phi) + n(\psi)$
- $s(Rt_1 \ldots t_m) = \bigcup_{1 \leq i \leq m} \{j \mid t_i \equiv \mathtt{p}_j\} \quad s(\neg\phi) = s(\phi) - n(\phi)$
  $s(\exists x\phi) = s(\phi) + 1 \quad s(\phi \wedge \psi) = (s(\phi) + n(\psi)) \cup s(\psi)$

Definition 2 (*PLA*: Models and Terms)

- a model $M = \langle D, E \rangle$ consists of a domain of individuals D and an interpretation E for the non-logical constants
- the interpretation $[t]_{M,g,e}$ of a term t is defined relative to a model M, an assignment g and a sequence e of at least $r(t)$ individuals
  $[c]_{M,g,e} = E(c) \quad [x]_{M,g,e} = g(x) \quad [\mathtt{p}_i]_{M,g,e} = e_i$

Satisfaction of a formula ϕ is defined relative to a model $M = \langle D, E \rangle$, assignment g, and sequence e of at least $r(\phi)$ individuals:

Definition 3 (*PLA*: Satisfaction)

- $M, g, e \models Rt_1 \ldots t_m$ iff $\langle [t_1]_{M,g,e}, \ldots, [t_m]_{M,g,e} \rangle \in E(R)$
  $M, g, e \models \neg\phi$ iff $\neg\exists d \in D^{n(\phi)}$: $M, g, d \cdot e \models \phi$
  $M, g, e \models \exists x\phi$ iff $M, g[x/e_1], e - 1 \models \phi$
  $M, g, e \models \phi \wedge \psi$ iff $M, g, e \models \psi$ and $M, g, e - n(\psi) \models \phi$
  (where $e - m$ is the sequence $e_{m+1}, e_{m+2}, \ldots$)

Observation 1 (Pronoun Resolution (Donkey Beating))

- $M, g, e \models \exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \wedge B\mathtt{p}_1\mathtt{p}_2$ iff
  $M, g, e \models \exists x(Fx \wedge \exists y((Dy \wedge Oxy) \wedge Bxy))$
- $M, g, e \models \exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow B\mathtt{p}_1\mathtt{p}_2$ iff
  $M, g, e \models \forall x(Fx \rightarrow \forall y((Dy \wedge Oxy) \rightarrow Bxy))$

Relative to an intensional model and an assignment function we can define the meaning of a formula as the set of pairs consisting of a world and a satisfaction sequence:

Definition 4 (Models and Meanings)

- an intensional model $\mathcal{M} = \langle W, D, I \rangle$ consists of a set of worlds, a domain of individuals and a world dependent interpretation function such that $\forall w \in W$: $\mathcal{M}_w = \langle D, I_w \rangle$ is an (extensional) model
- $[\![\phi]\!]_{\mathcal{M},g} = \{we \mid w \in W \ \& \ e \in D^{s(\phi)} \ \& \ \mathcal{M}_w, g, e \models \phi\}$

The (first order) information of linguistic agents is presented in so-called information aggregates, which consist of a domain (indicating the number of belief objects) and a set of world-assignment pairs:

Definition 5 (Information Aggregates)

- $\mathcal{P}(W \times D^n)$ is an information aggregate with n belief objects

The most important relation on an information structure is that of support and the most important operation is the product operation. Both notions are defined relative to functions l linking the belief objects of one aggregate to those of another:

Definition 6 (Information Product and Support)

- $\sigma \sqcap_l \tau = \{wee' \mid we \in \sigma \text{ and } wee' \in_l \tau\}$
- $\sigma \sqsubseteq_l \tau$ iff $l \in s(\sigma)^{s(\tau)}$ and $\forall we \in \sigma$: $we \in_l \tau$

(where $we \in_l \tau$ iff $we_{l(1)} \ldots e_{l(n(\tau))} \in \tau$)

The logical notions of product and support can now be used to characterize linguistic notions of update and licensing:

Definition 7 (Update and Licensing)

- $(\tau)[\![\phi]\!]_{M,g} = [\![\phi]\!]_{M,g} \sqcap_{+n(\phi)} \tau$
- $\sigma \models_{M,g,l} \phi$ iff $\sigma \sqsubseteq_l [\![\phi]\!]_{M,g}$

The notions of update and support are also defined independently in a compositional fashion. As an illustration, consider the clause for conjunction:

Observation 2 (Meaning, Update and Support of Conjunctions)

- $[\![\phi \wedge \psi]\!]_{M,g} = [\![\psi]\!]_{M,g} \sqcap_{+n(\psi)} [\![\phi]\!]_{M,g}$
- $(\tau)[\![\phi \wedge \psi]\!]_{M,g} = ((\tau)[\![\phi]\!]_{M,g})[\![\psi]\!]_{M,g}$
- $\sigma \models_{M,g,l} \phi \wedge \psi$ iff $\sigma \models_{M,g,l} \psi$ and $\sigma \models_{M,g,l-n(\psi)} \phi$