A formality

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17 March 1999

Abstract

Contrary to what I and other people have said in print, the phrase ‘formal logic’ has no traditional meaning. Kant very occasionally used the term. But it first came into regular use in the second half of the 19th century in Britain and New England, and it meant various things to various people. Appeals to ‘tradition’ in logic have a habit of being built on sand.

Contents

1 Formal logic?

This note is an apology.

In my youth I once used the phrase 'formal logic' in print as a name for the part of logic that is done in symbols—mathematical logic in fact. This provoked a complaint from a philosopher who told me that 'formal logic' has a historical meaning and I shouldn't try to change it.

Later I wrote ([13] p. 2):

> Traditional logicians collected valid argument schemas ... This activity used to be known as *formal logic* on the grounds that it was concerned with the forms of arguments. (Today we more often speak of formal versus informal logic, just as formal versus informal semantics, meaning mathematically precise versus mathematically imprecise.)

Nobody complained about that. Nevertheless it is this second statement that I am going to apologise for. One ought not to make historical statements in print without checking them, and I hadn't checked this one.

Recently I did start to check it, after becoming suspicious of some other claims made in the literature about 'traditional' notions in logic. I found that the chief mistake I had made was to think that the term 'formal logic' was traditional at all. In fact it has no historical connection with the 'consequences valid by reason of form' of the late medievals, as I had assumed. It first appeared with any regularity in the mid-to-late nineteenth century, and though some people have meant something fairly precise by it, the agreed and common content of the phrase is and has always been pretty thin.

I thank Maria Panteki for some valuable historical information, and particularly for alerting me to Immanuel Kant's role in the matter.

2 1781–1839, Kant

No doubt somebody somewhere used the phrase 'formal logic' before 1781. But the first appearance that I know of is in the first edition of Immanuel Kant's 'Critique of Pure Reason' ([15] p. A131), where he uses 'bloss formale Logik' ('merely formal logic') as a passing synonym for what he regularly calls 'allgemeine Logik' ('general logic') in that work.

There is very little here to get excited about. Kant uses the phrase almost incidentally. As far as I know, the phrase 'formale Logik' never occurs in all Kant's voluminous lecture notes on logic. (I have just checked through the Vienna-Hechsel notes from around 1780 [16].)

According to Kant, logic is the study of the laws of the understanding ('Verstand'). So his logic is closer to what we know as Theory of Knowledge than what we know as Logic, though in his logic notes he does make some rather perfunctory remarks about syllogisms. The laws may be different according to the subject matter; if one is interested in the laws of understanding X, one can call it X-logic. Thus Kant's 'transzendentale Logik' studies the laws of the understanding of a priori objects. Or to steal an example from De Morgan ([11] p. 218), fenestral logic is the study of the laws of the understanding of windows.

But logic, plain logic, studies the laws which apply to the understanding of just anything, so Kant calls it 'general logic' for emphasis. (Thus [15] p. A50ff.)

Why 'formal'? Because everything is a combination of form and matter, and if you remove the matter, what's left is form. So if you abstract from the subject-matter of the understanding, what's left is by definition the form of the understanding. This is Kant's usual terminology. (See for example [16] p. 251: 'In all thought there is matter and form. Matter concerns the object and form the mode of treatment. ... Thus a science that is occupied with the form of the understanding is called logic.')

So 'merely formal logic' is just logic. Probably the fairest answer to the question 'What does "formal" mean here?' is that it doesn't mean anything; it was put there to block any restriction of subject matter. Most emphatically it doesn't mean anything about forms of arguments or forms of propositions.

Commentators on Kant would naturally use his terminology, and inevitably some of them picked up this phrase 'formal logic'. I don't know of any commentator who used the phrase in print before 1839, when Thomas Solly of Walthamstow used the distinction Transcendental Logic versus Formal Logic in his 'A syllabus of logic' ([25], see also Panteki [20]). Panteki notes that this work 'enjoyed only a very limited circulation'. De Morgan only became aware of it just in time to mention it in a note added in proof to his book Formal Logic in 1847 ([6] p. 335f).

Another logician who used the phrase 'formal logic' in the course of discussing Kant's Critique was the young C. S. Peirce, in the opening words of the second draft of his paper 'On a new list of categories' [21] (probably 1862/3). But the phrase has disappeared in the published version [22] (1867).

It's depressing to note that twentieth-century commentators on Kant's Critique regularly refer to 'formal logic', although Kant barely ever used the phrase and his thoughts never came near what the phrase will suggest to most twentieth-century readers.

As far as I know, logicians before Solly didn't use the phrase. I couldn't find it in the lucid and politically incorrect 'Elements of Logic' by Richard Whately [27] (1826) or in the unintentionally hilarious 'Outline of a New System of Logic' by George Bentham [1] (1827). It is not in George Boole's 'Mathematical Analysis of Logic' [2] (1847). Nor is it in the first edition of John Stuart Mill's 'A System of Logic' [18] (1843).

But when Mill prepared the 1865 edition of his work, he added a section 'Of Formal Logic, and its relation to the Logic of Truth'. This new section, he explained, was about ([18] II.iii.9):

> the true nature of what is termed, by recent writers, Formal Logic ...

He gave no references, and the only writers he attributed the phrase to were (the Scottish) Sir William Hamilton and Whately. The reference to Whately seems to be a mistake (I checked the 1866 edition of Whately). But Mill was right about Hamilton.

3 1837–1856, Hamilton

William Hamilton lectured on logic at Edinburgh from 1837 till his death in 1856. He published very little on logic in his lifetime, but his lecture notes on logic (which apparently stayed more or less fixed over the years) were assembled and published in 1860 by Mansel and Veitch [12]. Mansel and Veitch also included as appendices some further notes which Hamilton never fully absorbed into the lectures, though he is known to have used parts of them in his teaching.

During his lifetime Hamilton had a reputation for erudition and inspirational lecturing. Reading the lectures today, it's hard to see why. They are not particularly erudite, even after the editors' efforts at packing out the footnotes with obscure references. (How could Hamilton completely miss his own 16th century predecessor at Edinburgh, the excellent John Major?) As for inspiration, they are practically content-free. In fact they form one vast catalogue of technical terms, with no visible guiding principles for the classification. Hamilton likes cataloguing so much that he even has an appendix (III, 'Divisions, varieties, and contents of logic') which is a catalogue of catalogues of logic.

I didn't find the phrase 'formal logic' anywhere in the lectures. But it does turn up in the appendices. I quote from Appendix I ([12] ii p. 231):

> The doctrine, therefore, which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the forms of thought, or in the conditions of the mind itself, which is the subject in which knowledge inheres,—this science may be called *Formal*, or *Subjective*, or *Abstract*, or *Pure Logic*. The science, again, which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the contents, materials, or objects, about which knowledge is conversant,—this science may be called *Material*, or *Objective*, or *Concrete*, or *Applied Logic*.

And from Appendix III ([12] ii p. 243):

> Perhaps, 1°, Formal Logic, (from the laws of thought proper), should be distinguished from, 2°, Abstract Logic, (material, but of abstract general matter); and then, 3°, A Psychological Logic might be added as a third part, considering how Reasoning, &c., is affected by the constitution of our minds. Applied Logic is properly the several sciences.

It's not easy to extract from these quotations anything that will add to what we already have from Kant. Though Hamilton hardly ever uses the phrase 'formal logic', he has some material in Lectures I and II about how 'the object of Logic is the formal laws of thought' (for example [12] i p. 28). It is very airy and I couldn't see that it adds anything.

The influence of Hamilton seems to have fallen away sharply after his death. I believe I was the first person to take out the two volumes of his logic lectures which Westfield College library presumably bought soon after their publication in the 1860s.

4 1846–1862, De Morgan

De Morgan's logical writings included five papers 'On the syllogism' ([5] 1846, [7] 1850, [8] 1858, [9] 1860, [10] 1862), in which he published his main original contributions, and a textbook 'Formal logic' [6] which came out in 1847. Curiously Mill's added material in his 1865 edition contained a quotation from De Morgan's 'Formal Logic' on the next page to his new section on formal logic, but he seems not to have noticed that De Morgan had the phrase.

In 1846 De Morgan uses the phrase 'formal logic' in section I of his first paper 'On the syllogism' [5], and again several times in a letter to William Whewell ([11] p. 195f). Each time he offers no definition, and he seems to assume that his readers will find the phrase familiar. The 1847 textbook contains no hint of an explanation for its title. In all these places but one, we get good sense if we read 'formal logic' simply as 'logic'. The one possible exception is the following paragraph to Whewell ([11] p. 196):

> Formal Logic usually is made only to treat of the copula. To be strictly *formal* I need not introduce ideal and objective, more than English and French, black and white, x and y. Two species of existence implied as belonging to the terms brought forward would do as well. But *ideal and objective* is the important distinction in practice, and as to assertion or denial, so far as I want it, is easy.

In 'On the syllogism II' [7] in 1850, De Morgan refers once to 'every kind of logic, formal and applied'. This still looks like Kant's distinction: logic itself versus X-logic, Y-logic etc.

By 1858 De Morgan felt that he should say more. Hamilton was now dead, and De Morgan seized the opportunity to do battle with his ghost. Sections 2 and 3 of the paper 'On the syllogism III' [8] are an attack on philosophers who wave the flag of 'form' but have no idea what it means. A Postscript in the paper identifies the culprits as chiefly the late Sir William, but also five other named philosophers 'and perhaps others'. Though De Morgan doesn't use the phrase 'formal logic' here, it's hard not to read the passage as a statement of what the title of his book means to him.

The central idea of the passage is that mathematicians have come to understand what the difference between form and matter is. (He probably has in mind recent work of Peacock, Gregory and himself on the nature of algebra.) I would have preferred it if De Morgan had simply told us what mathematicians think the difference is. But he would rather give us metaphors and examples. I go straight to one of his examples, from section 3 [8]:

> In the following chain of propositions, there is exclusion of matter, form being preserved, at every step:–

		Hypothesis
(Positively true)	Every man is animal	
—	Every man is Y	Y has existence
—	Every X has Y	X has existence
—	Every X—Y	— is a *transitive* relation
—	α of X—Y	α a fraction $<$ or $= 1$
(Probability β)	α of X—Y	β a fraction $<$ or $= 1$

> The last is *nearly* the purely formal judgment ... Strike out the word *transitive*, and the last line shews the pure form of the judgment.

Looking down this list, we see that nearly every phrase below the sentence 'Every man is an animal' in the second column is a pattern which has the phrase above it as an instance or special case. At a first glance, going down the list is *moving to more general patterns.*

Passing from line 1 to line 2 and from line 2 to line 3, we replace a phrase by a symbol with a condition on possible interpretations of the symbol. The condition is that the symbol should be read as referring to a non-empty class. (Note that because of the added condition, the higher lines are not just instances of syntactic patterns in the lower lines.)

The passage from line 3 to line 4 is meant to be similar: we replace 'is' by any transitive relation. Unfortunately this doesn't work: for example 'Every man = animal' is gibberish. De Morgan seems to have meant it as shorthand for something along the lines of

> Each X is in the relation R to some Y.

Then an instance might be 'Each man is in the relation of having the same birthday to some woman'. Note that in this case 'Every man is an animal' is not even an instance of the pattern; at best it is a paraphrase of an instance.

¿From line 4 to line 5 goes by replacing 'Each X' by 'A proportion α of the Xs'. Today we would raise an eyebrow at the assumption that a statement about every real number can be rewritten as a statement about 100% of the real numbers; but De Morgan was happy to assume that the universe of discourse was finite. The step from line 5 to line 6 doesn't seem to be a generalisation at all, unless his point is that we can meaningfully talk of probabilities in cases where we can't meaningfully talk of proportions.

There is no clear motive for grouping together these different kinds of generalisation. Also some of the details look arbitrary. For example, why must the relation in line 4 be transitive? The answer is clear from De Morgan's discussions here and elsewhere. In spite of appearances, De Morgan is not generalising forms of proposition. He is generalising *forms of argument.* A typical form of argument that he has in mind at line 4 is the syllogism in mood Barbara:

> Every X is a Y. Every Y is a Z. Therefore every X is a Z.

His point is that the following argument form also works:

> Each X is in relation R to some Y. Each Y is in relation R to some Z. Therefore each X is in relation T to some Z.

provided that R is transitive. (Actually he quotes the syllogism in mood Camestres to make his point, but the point is exactly the same.)

For the switch to proportion α, he has to choose his argument more carefully. He could instead have stayed with Barbara and made a quite different generalisation by passing to 'At time t, every X—Y', for example. For the final step, striking out '*transitive*', I can't imagine that he has any argument form in mind at all.

All I see here is a string of different kinds of generalisation, for different purposes. Some of them are generalisations of the form of the proposition, others are really generalisations of argument forms. In short it's a muddle. And it's certainly not what your average twentieth century logician understands by 'forms of propositions' or 'forms of arguments'.

5 1865–1898, Mill, Keynes, Peirce

In his addition of 1865, Mill said ([19] II iii 9):

> What, then, is Formal Logic? The name seems to be properly applied to all that portion of doctrine which relates to the equivalence of different modes of expression; the rules for determining when assertions in a given form imply or suppose the truth or falsity of other assertions.

This looks quite modern. But there is no evidence that Mill was doing anything more than guessing or legislating. Mill's book was very influential, and these words of his very probably helped to fix the meaning of the phrase.

De Morgan didn't even persuade his own student W. S. Jevons to use the phrase 'formal logic' ([14]). Boole generally avoided the phrase in the 424 pages of his 'Laws of Thought' [3] (1854); I found it only on pages 176, 204 and 227, and one of these is quoting the title of De Morgan's book. But De Morgan had better luck with J. N. Keynes, whose 'Studies and Exercises in Formal Logic' [17] of 1884 surely owe their title to De Morgan's.

De Morgan also had an eager disciple in Peirce, who already knew the term from Kant (as we saw). De Morgan's work on relations inspired Peirce to make an interesting innovation in 1873 (first page of [23]): Peirce proposed that there are various 'logics', including 'the logic of relatives' and 'the logic of absolute terms'. This latter logic, essentially what we know as boolean algebra, was in Boole's time 'the only formal logic known'. In later years Peirce used the phrase 'formal logic' to cover both what he himself did in logic and what Kant had discussed under that name. See for example the opening section of Lecture II from his Cambridge Lectures of 1898 [24]. Here he even offers a definition of sorts:

> The very idea of formal logic is, that certain *canonical forms* of expression shall be provided, the meanings of which expressions are governed by inflexible rules.

(Is it? What can he mean?)

It doesn't take long for the new to become the time-hallowed. Already in 1913 the Catholic Encyclopedia [4] told its readers that the classification 'formal logic' was 'traditional'.

6 To conclude

The expression 'formal logic' became a set phrase during the period 1839–1884. Probably De Morgan's use of it as the title of his book in 1847, and Mill's paragraph in his 1865 edition, did more than anything to fix it in people's minds. At first the term meant 'logic in general as opposed to the logic of this or that special area'. It was Mill in 1865 who first connected the phrase with argument forms and forms of propositions; though De Morgan's propaganda of 1858 may have prepared the way for this connection. The term was in regular use among English-speaking logicians from the early 1880s onwards, though not always with enough of a definition to tell us what it meant.

In 1936 Alfred Tarski [26] proposed to analyse 'the common concept of consequence', adhering as far as possible to 'the common usage of the language of everyday life'. At a key point he said:

> . . . we are concerned here with the concept of logical, i.e. *formal*, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds . . .

Many writers have attacked Tarski's analysis, and though some have repeated his view that the language of 'everyday life' includes a concept of 'formal consequence', I have never heard anybody defend this view. But most commentators have accepted that there was an agreed concept of formal consequence in the logical tradition, and that it meant what Tarski says in the passage quoted; the only issue was whether he correctly formalised this agreed concept. This amazes me. Wading through the 19th century literature, I see no trace of an agreed concept of the form of a sentence.

7 Envoi

Yes, I know it's inappropriate to dedicate a note on the history of logic to a logician with his eye so firmly fixed on the future as Johan van Benthem. But this time I don't apologise.

Twentieth century logic made astonishing progress by throwing in its lot with the mathematicians, as De Morgan hoped it would. Johan is a pioneer among those who believe that it is time to extend the mathematical techniques of pure logic into those areas where logic is applied. One side-effect, small but good, will surely be that the next generation of historians of logic has better tools for appreciating the achievements of earlier pioneers.

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