

# Historical mistakes

*Comments on the Fregean revolution in philosophy of language*

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## Abstract

The historic relation between Frege and ‘Frege’s principle’ is investigated. What did Frege intend with the principle, did he change his opinion concerning it, did he invent the principle? Johan van Benthem did twenty years ago research on another aspect of Frege’s work and some of his opinions are confirmed by the present research. His pregnant formulations and suggestive metaphors are quoted where possible

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## 1 Introduction

My research during the last year concerned the historical background of ‘Frege’s principle’: what was the relation between Frege and the principle, and what is the historic context of the principle. Recently I became aware of the fact that Johan van Benthem had done a comparable research twenty years ago concerning Frege’s ‘Begriffsschrift’. He discusses several issues which also arose in my research, and came with related conclusions twenty years before. On one point he even gives an argument that is missing in my draft. The formulations Johan uses are very pregnant, with unusual metaphors, and I regret that they are only published in a philosophical journal in Dutch [2]. Here I will present a summary of my research, and quote Johan’s formulations where appropriate. The title of the present contribution is a variant of the title of his article.

## 2 The Problem

The principle of ‘compositionality of meaning’ reads in a well known formulation [11]:

The meaning of a compound expression is a function of the meaning of its parts and of the syntactic rule by which they are combined

This principle is often called, in linguistics, logic and philosophy of language: ‘Frege’s principle’. However, in a distinct philosophical community, another principle is known as ‘Frege’s principle’, viz. the ‘principle of contextuality’. A picture of the two communities is given by Pelletier [12]. This principle of contextuality occurs in in ‘Grundlagen der Arithmetik’ [5, p.x]:

nach der Bedeutung der Wörter muss im Satzzusammenhange, nicht in der Vereinzelung gefragt werden <sup>1</sup>

If one takes the two principles literally, just as they are formulated, there is conflict. Compositionality requires that there is a function that takes as inputs meanings of parts. It is not possible to have a function that takes as inputs elements from a domain that does not exist. Hence, there has to be a domain of such meanings. Consequently, these meanings have to exist outside the context of the sentence in which they occur. And that precisely is what contextuality forbids: meanings of expressions in isolation. So it seems that not both can be ‘Frege’s principle’, unless at some moment Frege has changed his opinion.

But is this black-and-white interpretation necessary? Or is an interpretation possible which reconciles the two principles? If one reads the principles less literally, than that indeed is the case.

The most influential proposal to reconcile the principles is by Dummett in his huge book on Frege’s philosophy of language ([3] pp.192–196). His view, summarized very briefly, is as follows. Contextuality expresses that it makes no sense to consider the meaning of word in isolation, and then go to some unrelated other issue. Speaking about the meaning of word only makes significance

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<sup>1</sup>Never ask for the meaning of a word in isolation, but only in the context of a proposition [6]

as preparation for considering the meaning of a sentence. However, Dummett does not base his interpretation on writings by Frege. Therefore one might ask whether it is indeed Frege's philosophy of language, or rather Dummett's.

### 3 Grundlagen

Since 'Grundlagen' der Analysis' [5] is the source of an explicit formulation of contextuality, it is interesting to see what Frege uses the principle for. It turns out that he gives several examples where we cannot associate a concept with a word, so where a word has no meaning in isolation. Below a paraphrase is given (using the translation in [6]) of his argumentation about infinitely small numbers (§60)

Frege argues as follows. 'Even a concrete thing as the Earth we cannot imagine as we know it to be.' It is too large, there is no way to have a conception of it. Such words, 'for which we can find no mental picture appears to have no content.' But 'that we can form no idea of the content of a word is not a reason for excluding it from our vocabulary'. 'It is enough that the sentence in which the word occurs as a whole has a sense'. This approach may clarify difficult concepts, such as infinitely small units. These arise in analysis, e.g in the identity  $df(x) = g(x)dx$ . 'The problem is not, as might be thought, to produce a segment bounded by two distinct points whose length is  $dx$ . 'But we ought always to keep before our eyes a complete proposition. Only in a proposition have the words really a meaning.' So we should define the sense of the identity of the type  $df(x) = g(x)dx$  as a whole.

This illustrates that Frege meant what he said: no meaning for words in isolation. And it shows that Dummett's explanation (mentioned in section 1) is Dummett's philosophy, and not Frege's. Johan describes this metaphorically by:

Over ieder woord uit zijn [= Frege's (T.J.)] kleine oevre lijkt het zinnige commentaar wel gepubliceerd (en vaak meer dan dat). Zo vallen de dunne taal filosofische geschriften in het niet bij de kloeke monografie [3], waarin de schrijver zijn hoofdrolspeler duchtig laat buiksprecken.<sup>2</sup>

### 4 Significant information

Did Frege change his opinion? An exciting story can be told about this based upon 'Booles rechnende Logik und die Begriffsschrift', one of Frege's posthumous articles. In his review [14] of the publication of Frege's posthumous works, Sluga points to that article in order to show that Frege in his later period still adhered contextuality. There we find the following quote ([9] p.18):

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<sup>2</sup>On each word of his [= Frege's (T.J.)] little oevre the sensible comments will have been published (and often more than that). The thin articles on philosophy of language pale before the sturdy monograph [3] in which the author makes his leading actor ventricle soundly.

Statt also das Urteil aus einem Einzeldinge als Subjecte mit einem schon vorher gebildeten Begriffe als Praedicate zusammen zu fügen, lassen wir umgekehrt den beurteilbaren Inhalt zerfallen und gewinnen so den Begriff.[...] Daraus folgt aber nicht, dass losgelöst von den Dingen die Vorstellungen dieser Eigenschaften und Beziehungen gebildet werden; sondern sie entstehen zugleich mit dem ersten Urteile, durch das sie Dingen zugeschrieben werden. Daher treten ihre Bezeichnungen in der Begriffsschrift nie vereinzelt auf, sondern immer in Verbindungen, welche beurteilbare Inhalte ausdrücken. Ich möchte dies mit dem Verhalten der Atome vergleichen, van denen man annimmt, dass nie eins allein vorkommt, sondern nur in einer Verbindung mit andern, die es nur verlässt um sofort in eine andere einzugehen. <sup>3</sup>

Resnik ([13] p.47) accepts it as a formulation of the context view, but says ‘that it is clearly not relevant because it was written *before* the *Grundlagen*’. Indeed ‘Booles rechnende Logik’ is from 1880 so 4 years before ‘Grundlagen’ (Frege several times submitted the paper in vain). In spite of this earley year of creation, the paper turns out to be highly relevant.

The article has a footnote on the last sentence of the quotation, in which Fregehe says that one can find the same picture in Wundt’s book ‘Logik’: ‘Dieses Bild had *Wundt* in seiner Logik, wie ich nachträglich gesehen habe, in ähnlicher Weise angewendet’ [16, p.95]. <sup>4</sup> The text of Wundt will be considered in detail in Section 5. Now the interesting aspect is that the authors of the original German edition of Frege’s ‘Posthumous writings’ inform us that this picture of parts as atoms does *not* occur in the first edition from 1880 of Wundt’s publication, but firstly in the 3th edition of 1906. Hence Frege must have added the footnote after 1906, which shows that he still accepted the contextual point of view in 1906. So the passage is relevant to illustrate Frege’s that Frege held contextuality many years after ‘Grundlagen’.

In the English translation the editorial footnotes of the German edition are not incorporated. However, this does not explain that Resnik misses the issue, because he presents another translation than the one in the English edition (not completely accurate, his translation of the comparison with the behavior of atoms omits ‘sofort’/‘immediately’).

## 5 Historical context

Johan characterizes a common perspective on history as follows [2, p.1]

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<sup>3</sup> And so, instead of putting a judgment together out of an individual as subject, and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgment.[...] But it doesn’t follow from this that the ideas of these properties and relations are formed apart from objects: on the contrary they arise simultaneously with the first judgment in which they are ascribed to things. I could compare this with the behavior of the atom: we suppose an atom never to be found on its own, but only combined with others, moving out of one combination only in order to enter immediately into another. [8, p.17]

<sup>4</sup>As I have since seen, Wundt makes a similar use of this image in his Logik. [8, p. 17]

Een gangbare voorstelling van zaken wil dat de logische Middeleeuwen in Europa duurden tot rond 1900 - en in Nederland dus tot 1950. Na een eeuwenlange 'tragedy of errors' was toen eindelijk de kalme toorts der rede ontstoken in Gottlob Frege's publicatie [4] . . . Geach spreekt in zijn inaugurele rede [7] zelfs van de noodzakelijke strijd tegen het 'Kingdom of darkness'.<sup>5</sup>

Johan did not agree with this perspective on history, and below we will see that his opinion is confirmed by the history of contextuality and compositionality.

### **Trendelenburg**

Trendelenburg was an influential German philosopher in the middle of the 19th. century. The quote below is from the third edition of his 'Logische Untersuchungen' [15] from 1870, so fourteen years before 'Grundlagen'. There he describes both the contextual process as well as the compositional process ([15] ch.XXIV p.533-534):

Da nun eine solche Gemeinschaft zwischen Denken und Sein besteht, so können nicht bloss die Dinge den Gedanken bestimmen, dass er sie geistig im Begriffe nachbilde, sondern auch der Gedanke die Dinge, dass sie ihn leiblich darstellen. Da ist der Gedanke vor der Erscheinung, und die Theile stammen aus dem vorgebildeten Ganzen, nicht, wie sonst, aus Theilen das Ganze.<sup>6</sup>

In another passage he speaks about the two possible directions of the relation between concepts and judgments. He mentions in one and the same sentence both contextuality and compositionality. ([15] ch.XXIV p.536)

Wie im Sein aus der Thätigkeit die Substanz hervorgeht und wiederum aus der Substanz Thätigkeiten: so werden aus Urtheilen Begriffe, aus Begriffe Urtheilen.<sup>7</sup>

This shows that long before 'Grundlagen' the idea's of compositionality and contextuality were known.

### **Lotze**

H. Lotze wrote in 1874 a book on logic [10] of which Heidegger in 1912 said: 'the fundamental book of modern logic'. He argues against psychologism; according

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<sup>5</sup>A current impression of things says that the logical middle ages in Europe lasted until round 1900 - hence in the Netherlands until 1950. After a centuries long 'tragedy of errors' then the calm torch of reason finally was kindled in Gottlob Frege's [4] . . . Geach speaks in his inaugural address [7] even about the necessary struggle against the 'Kingdom of darkness'.

<sup>6</sup>Since there is such a unity between thinking and being, it is not only the case that things determine a thought in such a way that the thought reconstructs in the mind with a concept. Also the thought can determine things, in such a way that they represent it physically. In that case the thought exists before the representation, and the parts arise from the previously formed whole, and not, as in other cases where the whole comes from the parts.

<sup>7</sup>Like in reality the substance originate from the activity, and the activity from the substance, concepts originate from judgments, and judgments from concepts.

to him logic deals with the world of objective idea's. The notion of objectivity applies directly to whole propositions, but only indirectly to concepts. Mistaken (ontological) doctrines of concepts have their origin in the separation of the concept from the propositional context. It is a warning one also finds with Frege [5, p. x]: separating something from the propositional context, is a source of errors. This doctrine had its ultimate roots in the thought of Kant. Kant had argued against the theory of ideas that judgments are formed out of previously given constituents, and that they possess an initial transcendental unity out of which we gain concepts by analysis. By the late nineteenth century the doctrine had become a standard argument. Wundt wrote (in 1920): 'It had become the dominating characteristic of logic and has in many respects remained so until today to regard the judgment as the beginning of all logical thinking from which the concept was supposed to originate through analysis'.

Did Frege know the work of the Kant? Johan considers the same issue concerning Begriffsschrift and says:

Frege zegt raadselachtig weinig over zijn logische verleden. Wellicht niet zo bekend is dat in Begriffsschrift impliciet een beroemde voorganger wordt besproken. Voorafgaande aan zijn introductie der logische operatoren loopt Frege namelijk Kant's tabel der oordeelsvormen na - overigens zonder dit expliciet te vermelden: deze behoorde destijds nog tot de algemene ontwikkeling. <sup>8</sup>

This table concerns the four types of judgment: Quantity, Quality, Relation, and Modality, each consisting in 3 subclasses. On all of them Frege gives his comments, in defense of his own approach.

As Johan showed, Frege did not have the habit to give references for common knowledge. Therefore it is not significant that he does not refer to Kant (or Lotze) when he mentions contextuality. The idea that judgments are primary was, well known, therefore Frege knew this, and it explains why Frege did not see a reason to give any arguments for his principle of contextuality.

## Wundt

The book of Wundt wrote in 1880 an important book on logic. I encountered it when I followed the trace concerning the Frege's footnote in 'Booles regnende Logik'. Before that moment I never had heard about Wundt, but Johan knew his work twenty years ago. He says about it:

Anders dan wel eens wordt gedacht stuitte de moderne logica in klassieke logische milieus niet altijd op blinde vooroordelen. Zo trachtte de invloedrijke logicus en psycholoog Wilhelm Wundt reeds Boole's gedachtegoed in zijn werk te incorporeren <sup>9</sup>

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<sup>8</sup>Frege says mysteriously little about his logical history. Maybe it is not so well known that in Begriffsschrift a famous predecessor implicitly is discussed. Preceding to his introduction of logical operators, Frege checks up Kant's table of forms of judgments - without mentioning this explicitly though: the table belonged to general education in those days.

<sup>9</sup>Opposite to occasional thoughts, modern logic did not always encounter blind biases in classical environments. Already the influential logician and psychologist Wundt tried to incorporate Boole's body of ideas.

The same appeared to hold for Wundt's opinion on contextuality and compositionality. Although he accepted Kant's arguments, he defends the new developments. His starting point is ([16] p.94).

Das wirkliche Denken besteht in Urteilen, und losgelöst vom Urteil hat daher der Begriff keine Existenz, ebenso wenig wie das einzelne Wort, das als Begriffszeichen dient, in der lebendigen Sprache anders als im Zusammenhang des Satzes Wirklichkeit besitzt. <sup>10</sup>.

This is a formulation of contextuality, used to say the same as Frege did. The quote could have been Frege's words.

In the next fragment ([16] p.95) Wundt shifts towards a modern point of view.

Diese Tatsache, daß die logischen Begriffe nicht ursprünglich selbständig gegebene Denkinhalte, sondern Zerlegungsprodukte der Urteile sind, hat manche Logiker veranlaßt, der Untersuchung des Begriffs die des Urteils voranzustellen. Sobald einmal die freilich immer noch verbreitete Meinung beseitigt ist, daß das wirkliche Denken in einer Verbindung ursprünglich selbständig existierender Begriffe oder Vorstellungen bestehe, wird man jedoch dieser Frage kaum einen anderen als einen didaktischen Wert zugestehen können. <sup>11</sup>

So, although Wundt completely agrees with the contextual principle, he nevertheless does not appreciate the approach based on research of thoughts. His reasons for this modern approach are given in the next passage; it contains the analogy with chemical elements, to which Frege gave a reference (see Section 4).

Die Logik ist also hier offenbar in der nämlichen Lage wie andere Gebiete, die zu analogen Abstraktionen genötigt werden. Aus den selben Gründen, aus denen die Grammatiker, obgleich das Wort so wenig wie der Begriff isoliert vorkommt, doch der Satzbildung die Wortbildung, oder der Chemiker, obgleich die chemischen Elemente nur in Verbindungen vorkommen, den Eigenschaften der Verbindungen die der Elemente voranstellt, wird auch für den Logiker diese Ordnung der Gegenstände die zweckmäßigere sein. Die alte methodische Regel, daß man von dem Einfachen ausgehen müsse, um das Zusammengesetzte zu verstehen, bewahrt eben auch da ihre Geltung, wo uns in Wahrheit in der wirklichen Erfahrung immer nur das Zusammengesetzte gegeben ist. <sup>12</sup>

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<sup>10</sup> The real thinking consists of thoughts, and separated from a thought a concept does not exist. The same holds for words in living language when used as a sign for a concept; they have reality only in the context of a sentence

<sup>11</sup>The fact that logical concepts are not independently given, but are obtained from judgments, was a reason for many logicians to give the investigation of judgments preference over the investigation of concepts. As soon as we get rid of the still widespread opinion that real thinking consists of a linking of originally independently existing concepts or representations, one will give this question hardly any other value than a didactic one.

<sup>12</sup>Logic is here in the same situation as other branches of science which are compelled to similar abstractions. Although a word does not occur in isolation, grammaticians study the

So Wundt uses here the picture of parts as atoms like Frege, but *not* to convey the same message as Frege. Whereas Frege uses it to emphasize contextuality, Wundt uses it to defend the compositional approach as a necessary abstraction of science.

## 6 Conclusions

The picture that emerges from these investigations is as follows. The ideas of contextuality and compositionality were known before Frege. His aim was to defend the objective truth of mathematics, and he based the argumentation in *Grundlagen* on a suitable known idea: contextuality. He fully agreed with contextuality in its strict interpretation, and he adhered to the basic idea of contextuality during his whole life. So calling one of principles ‘Frege’s principle’ is historically not correct, and must be a later form of honoring him. Therefore I agree with the Johan’s words

In de vijftiger en zestiger jaren werd Frege van een obscuur Duitse kamergeleerde postuum verheven tot kerkvader van de moderne logica.<sup>13</sup>

The same holds for modern philosophy of language.

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formation of sentences from words. Although chemical elements only occur in compounds, chemists study the properties of elements. And also for logicians it will be better follow the same order. The old methodological rule that one has to start with the simple, in order to understand the combination, still has its value, even when it is true that in real experience only the combination is given.

<sup>13</sup>In the fifties and sixties, Frege was proclaimed from an obscure German scholarly recluse to a Church Father of modern logic



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