# Tolstoy's Mathematics in "War and Peace"

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**Abstract**

The nineteenth century Russian author Leo Tolstoy based his egalitarian views on sociology and history on the then newfangled statistical physics and also proposed a mathematical theory of waging war.

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## 1 Introduction

It is interesting to consider the excursions of mathematicians and scientists into prose and poetry, and conversely and less known, the explorations of poets and novelists into mathematics.

An example of the first is Luitzen E.J. Brouwer's excursion into literature and environmentalism [1], an appeal *avant la lettre* to save the earths natural environment from human polution. In particular he wants to abolish the technology that enables man's supremacy over nature and the physics and mathematics that makes this possible. Only pure ('intuitionistic') mathematics that by its nature is unapplied and unapplicable for evil purposes, and which is the ultimate creation of the noble mind, should be saved.

In another direction, the great Russian mathematician Andrei N. Kolmogorov was particularly interested in the form and structure of the poetry by the Russian author Pushkin [3]. He also remarks [4]: "what real meaning is there, for example, in asking how much information is contained in 'War and Peace'? Is it reasonable to include this novel in the set of 'possible novels,' or even to postulate some probability distribution for this set? Or, on the other hand, must we assume that the individual scenes in this book form a random sequence with 'stochastic relations' that damp out quite rapidly over a distance of several pages?

The answer to the latter question is decidly 'no'. There is a ubiquitous general theme in 'War and Peace', namely, the idea that single individuals cannot influence in any sense the course of history (contrary to what is assumed in common history writing), but that the course of history is determined by the confluence of myriad motions of the infinitesimally small individual human acts of free will, much as a flock of birds wheels about in unision without any apparent governor. Here we have individual humans as interchangeable atoms of ideal gas that in combination determine effects on macroscopic scales such as heat and pressure, following the nineteenth century statistical physics of H. von Helmholtz. It serves to justify egalitarian doctrine. Helmholtz is also the author of the unrelated whitticism, so true and so unknown to politicians and managers of science: " Whoever in the pursuit of science, seeks after immediate practical utility may rest assured that he seeks in vain," [2].

The author of 'War and Peace,' the great Russian novelist Count Leo Nikolayevich Tolstoy, had an intense interest in mathematical approaches to the sciences, as appears from his proposals to found sociology, history, and the science of war as a mathematical discipline, much like mathematician John von Neumann proposed to found the science of economy as a mathematical discipline in [6].

Tolstoy's views on the matter are set forth at great length in 'War and Peace', by many regarded as the greatest novel in any language. Based on, or perhaps called upon as justification for, Tolstoy's egalitarian philosophy, it is set forth passionately in long interludes littered through the later parts of this great novel. Recall that the book is ostensibly about the doings and adventures of a group of aristocratic people, and in the descriptions of great battles, at the time of Napoleon's invasion in the bleak reaches of great Russia. Closer inspection reveals that one of the main themes of the tale is the insignificance and expendability of the particular heros—like Napoleon—in the sweep of history: the events would have infolded in the same way irrespective of the so-called main figures. We base our treatment on Rosemary Edmonds 1957 translation into English published in Penguin Classics[5] (part I, 1972 printing; part II, revised 1978 printing). I will refer to the page numbers as [WP, xx].

## 2 Mathematical Sociology

Tolstoy disagrees with the view of history that ascribes the evolution of events to individuals:

"One might have supposed that the historians, who ascribe the actions of the masses to the will of one man, would have found it impossible to fit the flight of Napoleon's armies into their theory, considering that during this period of the campaign [in Russia] the French did all they could to bring about their own ruin, and that not a single movement of that rabble of men ... betrayed a hint of rhyme or reason. But no! Mountains of volumes have been written by historians ... [with] accounts of Napoleon's masterly arrangements and deeply considered plans ..." [WP, 1266]

Not only that individuals cannot be the main governors of the making of History, but:

"It is beyond the power of the human intellect to encompass *all* the causes of a phenomenon." ... "the human intellect ... snatches at the first comprehensible approximation to a cause and says: 'There is the cause'." Tolstoy goes on [WP, 1168] to explain "in historical events (where the actions of men form the subject of observation) the primeval conception of a case was the will of the gods, succeeded later on by the will of those who stand on the historical foreground—the heroes of history."

On page [WP, 1342] Tolstoy continues to unmask common misconceptions of traditional views of History:

"Why did things happen thus, and not otherwise? Because they did so happen. '*Chance* created the situation; *genius* made use of it,' says history. But what is *chance*? What is *genius*? The words *chance* and *genius* do not denote anything that actually exists, and therefore they cannot be defined. These two words merely indicate a certain degree of comprehension of the phenomena. I do not know why a certain event occurs; I suppose that I cannot know: therefore I do not try to know, and I talk about *chance*. I see a force producing effects beyond the scope of ordinary human agencies; I do not understand why this occurs, and I cry *genius*."

Now we come to the true view of history, in the spirit of the so successful natural sciences. The "unreasonable effectiveness of mathematics in science" as phrased by E. Wigner, must be extended *avant la lettre* to sociology and political history [WP, 977]:

"To elicit the laws of history we must leave aside kings, ministers, and generals, and select for study the homogeneous, infinitesemal elements which influence the masses. No one can say how far it is possible for a man to advance in this way to an understanding of the laws of history; but it is obvious that this is the only path to that end, and that the human intellect has not, so far, applied in this direction one-millionth of the energy which historians have devoted to describing the deeds of various kings, generals and ministers, and propounding reflections of their own concerning those deeds."

How then is this proper view of history obtained? Tolstoy discusses the continuity of motion that was captured in laws by dividing continuity into units. He observes that this can be done in a wrong way, [WP, 974]:

"Take, for instance, the well-known sophism of the ancients which set out to prove that Achilles would never catch up with the tortoise that had the start on him, even though Achilles traveled ten times as fast as the tortoise: by the time Achilles has covered the distance that separated him from the tortoise, the tortoise has advanced one-tenth of that distance ahead of him. While Achilles does this tenth the tortoise gains a hudredth, and so on *ad infinitum*. This problem appeared to the ancients insoluble. The absurdity of the finding (that Achilles can never overtake the tortoise) follows from arbitrarily separating the motion into separate units, whereas the motion of Achilles and the tortoise was continuous.

By adopting smaller and smaller units of motion we only approximate the solution of the problem but never reach it. It is only by admitting infinitesimal quantities and their progression up to a tenth, and taking the sum of that geometrical progression, that we arrive at the solution of the problem."

Now we come to the heart of the matter: Tolstoy's proposal of a differential and integral analysis of history [WP, 974–975]:

"A new branch of mathematics, having attained the art of reckoning with infinitesimal, can now yield solutions to other more complex problems of motion which before seemed insoluble. This new branch of mathematics, which was unknown to the ancients,[1] by admitting the conception, when dealing with problems of motion, of the infinitely small and thus conforming to the chief condition of motion (absolute continuity), corrects the inivitable error which human intellect cannot but make if it considers separate units of motion instead of continuous motion. In the investigation of the laws of historical movement precisely the same principle operates.

The march of humanity, springing as it does from an infinite multitude of individual wills, is continuous. The discovery of the laws of this continuous movement is the aim of history. But to arrive at these laws of continuous motion resulting from the sum of all those human volitions, human reason postulates arbitrarily, separated units. The first proceeding of a historian is to select at random a series of successive events and examine them apart from others, though there is and can be no *beginning* to any event, for an event flows without break in continuity from another. The second method is to study the actions of some one man—a king or a commander—as though their actions represented the sum of many individual wills; whereas the sum of the individual wills never finds expression in the activity of a single historical personage.

. . . Only by assuming an infinitesimal small unit for observation—a differ-

[1] Apart from Archimedes and Eudoxos [PV].

ential of history (that is, the common tendencies of men)—and arriving at the art of integration (finding the sum of the infinitesimals) can we hope to discover the laws of history."

## 3 Mathematics of War

The causality involved in war defies simple analysis, Tolstoy says, but is reached by the integration of the infinitesimal individual causes, [WP, 1184]:

"An infinite amount of freely acting forces (and nowhere is a man freer than during a life and death struggle) influence the course taken by a battle, and that course can never be known beforehand and never coincides with the direction it would have taken under the impulsion of any single force.

If simultaneously and variously directed forces act on a given body, the direction which that body will take cannot be the course of anyone of the forces individually—it will always follow an intermediate, as it were, shortest path, or what is presented in mechanics by the diagonal of a parallelogram of forces."

In [WP, 1223—1224] Tolstoy outlines the mathematics of war and goes into an explicit calculation that is patently false:

"Military science says, the greater the numbers [of an army] the greater the strength. ... For militray science to make this assertion is like defining energy in mechanics by reference to the mass only. It is like saying that the momenta of moving bodies will be equal or unequal according to the equality or inequality of their masses. But momentum (or 'quantity of motion') is the product of mass and velocity. So in warfare the strength of an army is the product of its mass and of something else, some unknown factor $x$."

He goes on to debate what this unknown $x$ may stand for and rejects the common explanations, especially the interpretation of $x$ as the amount of genius of the commanding general. He goes on to say that [WP, 1224]:

"We must accept the unknown and see it for what it is: the more or less active desire to fight and face danger. Only then, expressing the known historical facts by means of equations, shall we be able to compare the relative values of the unknown factor; only then may we hope to arrive at the unknown itself.

If ten men, batalions or divisions, fighting fifteen men, batallions or divisions, beat the fifteen—that is, kill or capture them all while losing four themselves, the loss will have been four on one side and fifteen on the other. Therefore, the four are equal to the fifteen, and we may write $4x = 15y$. In other words, $x$ is to $y$ as 15 is to 4. Though this equation does not yet give us the absolute value of the unknown factor, it does give us a ratio between two unknowns. And by putting a whole variety of historical data (battles, campaigns, periods of warfare, and so on) into the form of such equations, a series of figures will be obtained which must involve the laws inherent in equations and will in time reveal them."

This argument of Tolstoy is remarkable. He compares the loss of the conquering army with the total of the vanquished army—perhaps on the grounds that the vanquished army is totally lost. Testing the idea by inserting more extreme figures, such as that an army of 1.000.000 men beats a small army of

10 men, while the conquering army looses one man, we obtain the equation $x = 10y$. This means that the fighting spirit of the million-men army exceeded necessarily the fighting spirit of the minuscule ten-men army tenfold. The problem with Tolstoy's reasoning here is that he equates the ratio of the loss of the conquering army (irrespective of the size of the total army) and the total of the beaten army (however small) with the ratio of the fighting spirit of the beaten army and that of the conquering army. In our opinion this reasoning is hard to defend in general as is shown by substituting extrem numbers as above. The general drift of the argument is of course reasonable. Note that (contrary to the intention of the author) the variables $x$ and $y$ may contain the quality of the commanders (much as the quality of performance of a good symphony orchestra greatly depends on the quality of the conductor).

## 4 Conclusion

It is seldom the case that a great author deems fit to incorporate extensive discussions about mathematical foundations of social sciences in a major literary novel. It is much more common that scientists strive for literary redemption. Tolstoy is one of the rare examples of the former. In fact, he gives definite proposals to mathematize history, sociology, and the sciences of war in line with the rational inclination of the nineteenth century.

## References


[1] L.E.J. Brouwer, *Leven, Kunst en Mystiek*, Waltman, Delft, 1905. Enlish translation by W.P. van Stigt, *Notre Dame J. Formal Logic*, 37:3(1996), 391–431.

[2] H.L.F. von Helmholtz, *Academic Discourse*, Heidelberg, 1862.

[3] A.N. Kolmogorov, Statistics and probability theory in research into Russian poetry, *Proc. Symp. on complex investigation of artistic creation*, Nauka, Leningrad, 1964, 23.

[4] A.N. Kolmogorov, Three approaches to the quantitative definition of information, *Problems in Information Transmission*, 1:1(1965), 1–7.

[5] L/N. Tolstoy, *War and Peace*, 1869. (English translation by Rosemary Edmonds, first published in 1957 in Penguin Classics.)

[6] J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, Wiley, 1944.